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Measurement Good Practice Guide

Estimating Uncertainties in Testing

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BMTA
BRITISH MEASUREMENT AND TESTING ASSOCIATION

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Measurement Good Practice Guide No. 36

Estimating Uncertainties in Testing

An Intermediate Guide to Estimating and Reporting Uncertainty of Measurement in Testing

Keith Birch
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"At least two numbers are required if an experiment is to give a result and a measure of its reliability"
N C Barford, "Experimental Measurements, Precision, Error and Truth",
Addison-Wesley Publishing Company, Inc, London

Abstract: The generally accepted approach to evaluating and expressing uncertainties in measurements undertaken by testing and calibration laboratories is given in *The Guide to the expression of Uncertainty in Measurement*, first published in 1993 by ISO, Geneva. That document, frequently referred to as *The Guide* or the *GUM*, presents a comprehensive study of these principles which, if followed, will ensure that measurement uncertainty is calculated and stated in a consistent manner in all situations. It is, however, a complex document and it has been found necessary to supplement it with simplified guidance for specific fields of measurement.

This publication is one such guidance document that follows on from Stephanie Bell's *A Beginner's Guide to Uncertainty of Measurement*, developing treatment of the subject to a level intended for testing laboratories. It presents principles and guidance for the estimation of measurement uncertainty which are applicable to most fields of testing but is not intended for calibration laboratories, which normally need to have a greater depth of knowledge of these principles than is presented here. The principles and guidance given here are consistent with *The Guide* and with the requirements of *ISO/IEC 17025, General requirements for the competence of testing and calibration laboratories* where they relate to estimating and reporting uncertainty of measurements.

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Estimating Uncertainties in Testing

Contents

1 Introduction

1.1 This publication provides general guidance on the estimation of uncertainties across many fields of testing. It is aimed at ensuring an approach that is in line with the *Guide to the Expression of Uncertainty in Measurement* (reference 1, Bibliography). In this document this reference is referred to as *The Guide*.

1.2 In the interests of keeping to an intermediate level of presentation of the general descriptive format, special cases, exceptions and qualifying remarks have not been dealt with in any great detail. Reference should be made to *The Guide* or other sources listed in the Bibliography in those instances when it may be necessary to resolve special difficulties arising in specific tests. However, equations have been included to complement the text using the same symbols, format and nomenclature as in *A Beginner's Guide to Uncertainty of Measurement* by Stephanie Bell (reference 2, Bibliography).

1.3 Further suggested reading, giving more detail of the principles and practice of the estimation of uncertainties, is given in the Bibliography.

1.4 Definitions of terms used in this publication and the *Beginner's Guide* are listed in alphabetical order in the Glossary and defined terms are printed in bold at the first appropriate occurrence in the main body of the text.

1.5 It is important to distinguish between **measurement uncertainty**, which is a measure of the bounds within which a value may be reasonably presumed to lie, and **measurement error**, which is the difference between an indicated value and the corresponding presumed true value. The measurement error is a quantity which often can be evaluated and, from this knowledge, a correction to the measurement can be applied. However, the identification of an error and its subsequent correction may not be possible with exactitude and this inexactitude will of itself contribute to the measurement uncertainty.

1.6 Similarly, errors must be distinguished from mistakes or blunders. By their very nature mistakes and blunders cannot be quantified or allowed for as part of the measurement uncertainty.

1.7 A further common source of confusion is the use of the term **tolerance**. A tolerance is properly the limiting or permitted range of values of a defined quantity. The classic example is the tolerances (minimum and maximum) of a particular dimension of a manufactured component, usually set to ensure proper engagement with a mating part. To be able to measure the component and ensure that the tolerances are met, the measurement uncertainty must clearly be smaller than the tolerance. A minimum ratio of 1 to 4 is often recommended. The term **tolerance** is also applied to measuring instruments, when it is meant to indicate the acceptable range of values indicated by the instrument when set up and tested by the manufacturer. In this usage, the tolerance is one contribution to measurement uncertainty when using the instrument. It is important, when a tolerance associated with measurement is met, to understand and to properly interpret its intended use.

2 The reasons for estimating uncertainty

2.1 The estimation of the uncertainty of a measurement allows meaningful comparison of equivalent results from different laboratories or within the same laboratory, or comparison of the result with reference values given in specifications or standards. Availability of this information can allow the equivalence of results to be judged by the user and avoid unnecessary repetition of tests if differences are not significant.

2.2 The uncertainty of the result of a test needs to be taken into account when interpreting those results. For example, comparison of measurements from different batches of material will not indicate real differences in properties or performance if the observed differences could simply be within the range of the inherent variation in the test procedure. Similarly, a deviation from a specified value for the properties or performance of a product may not be significant if the difference between the specified value and the test result is within the range of uncertainty.

2.3 In some cases the uncertainty in a measurement or test result may be considered to be so small as to not merit formal evaluation. However, without a formal estimate, this consideration remains intuitive and, when challenged, a convincing response will not be possible.

2.4 The results of some types of test are subject to large uncertainty, for example where tests are carried out on samples that are themselves inconsistent in their properties. In such a case it might be asserted that even a relatively large uncertainty to be associated with the measurement may be ignored in comparison with the uncertainty due to sample variation. However, unless an estimate of the measurement uncertainty is made, the validity of this assertion cannot be supported.

2.5 An estimation, or at least a full consideration, of the components contributing to the overall uncertainty of a measurement or test result provides a means of establishing that the measurements made and the results obtained are valid. It may also help to confirm that tolerances included within a performance specification have been met or that the item under test is fit for the intended purpose.

2.6 The thorough assessment of the components contributing to the measurement uncertainty may also indicate aspects of a test method to which attention should be directed in order to improve procedures and accuracy of the measurement. It may also improve the understanding of the principles of the test method and practical experience of its application can be a key part of method validation.

2.7 Finally, with the implementation of International Standard ISO/IEC 17025, *General requirements for the competence of testing and calibration laboratories*, there now exists a general requirement for the estimation and reporting of uncertainty of measurement by all accredited laboratories. The standard requires that laboratories "shall have and shall apply a procedure to estimate the uncertainty of measurement". The degree of rigour needed in an estimation of uncertainty may differ between testing and calibration laboratories and in some cases the nature of the test method may preclude rigorous, metrologically and statistically valid calculation of measurement uncertainty. Nonetheless, the laboratory is required to attempt to identify all sources of uncertainty and make a reasonable estimate.

3 General principles

3.1 The objective of any measurement is to determine the value of the **measurand**, that is the specific quantity subject to measurement. A measurement should, therefore, begin with an appropriate specification of the measurand, the identification of the generic method of measurement in the form of a functional model, and a related measurement procedure.

3.2 In general, no measurement or test is performed perfectly and the imperfections in the process will give rise to error in the result. Consequently, the result of a measurement is, at best, only an approximation to the true value of the measurand and is only complete when the measured value is accompanied by a statement of the uncertainty of that approximation.

3.3 Errors in the result may be thought of as arising from two sources, a random component arising from variations in repeated measurements and a systematic component, as may arise from the imperfect correction of systematic effects. Evaluating the measurement uncertainty in any particular situation therefore comprises the identification, quantification and combination of these components.

3.4 Errors may arise from random variations of observations (random effects). Every time a measurement is taken under nominally the same conditions, random effects from various sources influence the measured value. A series of measurements therefore produces a scatter of values distributed around a **mean value**. A number of different sources may contribute to variability each time a measurement is taken, and their influence may be continually changing. They cannot be eliminated by the application of correction factors but the uncertainty in the mean value due to their effect may be reduced by increasing the number of observations.

3.5 Further errors arise from systematic effects, that is an effect on a measurement result of a quantity that may not have been included in the original specification of the measurand but nevertheless influences the result. These remain unchanged when a measurement is repeated under the same conditions and their effect is to introduce a bias or offset between the presumed true value of the measurand and the experimentally determined mean value. Systematic errors can be corrected but there may be some remaining uncertainty in the value of the applied correction.

3.6 The sources of error identified above have been grouped into random errors and systematic errors. The Guide then adopts the approach of grouping uncertainty components into two categories based on their method of evaluation, “Type A” and “Type B” respectively. This categorisation, based on the method of evaluation rather than on the components themselves, avoids certain ambiguities - a “random” component of uncertainty in one measurement may become a “systematic” component in another measurement that has as its input the result of the first measurement. For example, the overall uncertainty quoted on a calibration certificate for an instrument will include the component due to random effects but, when this overall value is subsequently used as the contribution in the evaluation of the uncertainty in a test using that instrument, the contribution would be regarded as systematic.

3.7 ***Type A* evaluation of uncertainty** is by a statistical calculation from a series of repeated observations. The statistically estimated standard deviation of those observations is called a *Type A* **standard uncertainty**. Note that it is sometimes appropriate to weight the estimated standard deviation by a **sensitivity coefficient**.

3.8 ***Type B* evaluation of uncertainty** is by means other than that used for *Type A*. For example, information about the sources of uncertainty may come from data in calibration certificates, from previous measurement data, from experience with the behaviour of the instruments, from manufacturers' specifications and from all other relevant information. *Type B* components are also characterised by estimated standard deviations which, when weighted by sensitivity coefficients, become *Type B* standard uncertainties.

3.9 All identified standard uncertainty components, whether evaluated by *Type A* and *Type B* methods, are combined to produce an overall value of uncertainty to be associated with the result of the measurement, known as the **combined standard uncertainty**.

3.10 To then meet the needs of industrial, commercial, health and safety, or other applications, it is usually required to convert the combined standard uncertainty to an **expanded uncertainty**, obtained by multiplying the combined standard uncertainty by a **coverage factor**, k. The expanded uncertainty provides a larger interval about the result of a measurement than the combined standard uncertainty with, consequently, a higher probability that the value of the measurand lies within that greater interval.

4 Sources of uncertainty

4.1 One of the most important aspects of uncertainty evaluation is the need for a detailed understanding of the measurement process and thence all potential sources of the measurement uncertainty. This may mean that the design engineer or skilled operator of the measurement system is best suited to perform the evaluation exercise. The identification of uncertainty sources begins by examining in detail the measurement process. This often includes a detailed study of the measurement procedure and the measurement system, using a wide variety of means, including flow diagrams and computer simulations, repeat or alternative measurements, intercomparisons with others.

4.2 The many possible sources of uncertainty in testing may include contributing uncertainties from some or most of the following:

- incomplete definition of the test; ie the requirement is not clearly described, for example the temperature of a test may be given as "room temperature";

- imperfect realisation of the definition of the test procedure; even when the test conditions are clearly defined it may not be possible to produce the required conditions;

- inadequate knowledge of the effects of errors in the environmental conditions on the measurement process;
- imperfect measurement of the environmental conditions;
- sampling; the sample may not be truly representative;
- personal bias in reading analogue instruments; parallax errors;
- instrument resolution, or the discrimination threshold, or errors in the graduation of the scale;
- values assigned to measurement standards (both reference and working) and reference materials;
- changes in the characteristics or performance of a measuring instrument since its last calibration; incidence of drift;
- errors in values of constants, corrections and other parameters used in data evaluation;
- approximations and assumptions incorporated in the measurement method and procedure;
- variations in repeated observations made under apparently identical conditions; such random effects may be caused, for example, by short-term fluctuations in local environment, eg temperature, humidity and air pressure, or by variability in the performance of the tester.

Note: It is important not to 'double count' uncertainty contributions. If an uncertainty component is included in the Type A determination, in should not be included as a separate Type B evaluation.

4.3 In addition to the sources indicated above, which are not necessarily independent, there may be other unknown systematic effects that cannot be taken into account but which, nonetheless, contribute to an error. The existence of such effects may sometimes be deduced, for example, from the examination of the results of an inter-laboratory comparison programme or by the use of a different measurement procedure.

4.4 In some areas of testing, notably in the analysis of chemical samples, individual sources of uncertainty cannot be readily identified, quantified and combined as outlined above. In such cases the evaluation of measurement uncertainty is carried out using data obtained during the development of the test method and from validation studies. The measurement uncertainty is then associated with the method and is not evaluated on each occasion the test is carried out. This approach to evaluation of measurement uncertainty in areas such as analytical measurements is discussed further in Appendix A, Method development and validation.

5 Estimation of uncertainties

5.1 General approach

5.1.1 At an early stage in the development of the uncertainty budget, it is necessary to model the measurement process by a functional relationship. This should identify all measured input quantities that will ultimately be combined to arrive at the value of the output quantity, or measurand, and should indicate the manner in which they are to be combined. In general terms, the functional relationship between the output quantity, y, and the estimated input quantities, x_i, is in the form

$$y = f(x_1, x_2, \ldots\ldots x_n)$$

but a more specific relationship should be identified where possible. This should indicate whether the output quantity is the sum or difference of the input quantities, whether there are products or quotients, powers of some input quantities or logarithmic quantities. The functional relationship should also indicate any correlated input quantities although in test measurements the incidence of such correlations is relatively infrequent.

In many instances of undertaking test measurements the functional relationship is relatively simple and often the test procedure provides the basis of a satisfactory model. The most frequent form of functional relationship is a linear combination of measurements,

$$y = (c_1.x_1 + c_2.x_2 + \ldots\ldots + c_n.x_n)$$

where the coefficients, c_i, termed **sensitivity coefficient**, may be a known coefficient, such as the temperature coefficient of expansion, or the partial derivative $\delta f/\delta x_i$. The important part played by the coefficients in the combination of standard uncertainties will become clear from later examples.

Examples may be taken from the measurement of electrical quantities. Resistance, R, may be measured in terms of voltage, V, and current, I, giving the functional relationship

$$R = f(V,I) = V/I$$

and Power, P, may be measured in terms of current, I, and resistance, R, giving the functional relationship

$$P = f(I,R) = I^2R$$

5.1.2 The total uncertainty of a measurement is found by the combination of all the contributing component uncertainties. Each individual instrument reading or measurement may be influenced by several factors and careful consideration of each measurement involved in the test is required, therefore, to identify and list all the factors that contribute to the uncertainty. This is the initial and most crucial step, requiring a good understanding of the measurement equipment, the principles of the test and the influence of the environment.

5.1.3 The next step, having identified the component uncertainties, is to quantify them by appropriate means. An initial approximate quantification may be valuable to enable some components to be shown to make an insignificant contribution to the total and not worthy of more rigorous evaluation. In most cases a practical definition of insignificant would be a component uncertainty of not more than one fifth of the magnitude of the largest component uncertainty.

5.1.4 Some components may be quantified by calculation of an estimated standard deviation from a set of repeated measurements (*Type A*). Quantification of others will require the exercise of judgement using all relevant information on the possible variability of each factor (*Type B*). These alternative methods are explained in greater detail in the two following Sections.

Note: To simplify calculations, all components uncertainties should be expressed in the same way, ie either in the same units as used for the reported result or as a proportion (percent, or parts per million, etc) of the measurand.

5.2 'Type A' method of evaluation

5.2.1 If a number of tests are made upon one or a number of identical samples under identical conditions, the measured values are not necessarily the same. Due to a variety of causes, such as electrical or thermal noise, vibrations, etc., the measurements may differ and be distributed about a mean value. If this random effect is significant in relation to the other components of uncertainty, it must be quantified and subsequently included in the determination of the combined uncertainty.

5.2.2 The effect described in 5.2.1 requires a *Type A* evaluation. That is to say, it must be evaluated by a statistical method. In practice, this is by a straightforward arithmetical calculation based upon the results of a number of similar measurements.

5.2.3 A number of measurements 'n' are made under the conditions of the test and are assumed to be normally distributed. The standard deviation of the measured results may then calculated from the measured values by the method given in Table 1.

5.2.4 If a test to be made for a client is one which is not regularly carried out by the laboratory, this component of uncertainty will need to be evaluated as part of that test. However, laboratories often carry out tests of an essentially similar nature on a regular basis, using the same equipment under similar conditions. In such circumstances it is usual for a special series of tests to be carried out to quantify this component of uncertainty, which is then assumed to be applicable to all similar tests.

5.2.5 The estimated standard deviation, s_{est}, is an estimate of that for the full population, $\sigma(x)$, of the random variable, x, as based on a limited sample of n results.

Table 1: Evaluation of mean and the estimated standard deviation

Description of operation	Mathematical formulae
3.1 Apply all corrections to the measured results. (See Note 1)	$x_i = (corrected\ measured\ result)$
3.2 Calculate the mean value of the corrected measured results by summing them and dividing by the number of results 'n'. (Apply the corrections if omitted in 3.1)	$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$
3.3 Subtract this mean value from each result in turn to give the residual for each measured result.	$Residual = (x_i - \bar{x})$
3.4 Square each residual, sum the squares of the residuals and divide by the number of results less one (n-1). The result is called the Variance.	$V = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{(n-1)}$
3.5 Take the positive square root to give the estimated standard deviation of the set of measured results (Apply the corrections if omitted in 3.1)	$s_{est} = \sqrt{V}$

Note 1: If these corrections, such as the conversion of units of measurement, are identical for each measurement, this step may be omitted and the correction applied to the mean value (3.2) or to the estimated standard deviation which will be derived (3.5).

5.2.6 The number of measurements made for a Type A evaluation which is to be used as a contribution to the uncertainty of later routine tests, should be sufficient to adequately characterise the standard deviation. This should be normally be at least 10 measurements, although fewer may have to be accepted if this number is not practicable (eg due to a lack of suitable specimens, difficulty or expense of the tests).

5.2.7 When only one measurement is then made during a test undertaken for the client, the standard deviation found in Table 1 should be used as the contribution of standard uncertainty.

Example; *Establishment of expanded uncertainty from a single measurement*

This example establishes the total expanded uncertainty derived from a single measurement from a previously calibrated machine.

During preliminary evaluation of the repeatability of operation of a Rockwell hardness operating system, 10 indentations were made on a Rockwell hardness test block with the following readings:

45.4 45.5 45.4 45.3 45.5 45.3 45.3 45.4 45.4 45.4 HRC

The related statistical parameters are: Mean value 45.39 HRC
Estimated Standard deviation ±0.074 HRC

For a single reading made with the same system on a subsequent occasion, the contribution to the standard uncertainty is the pre-estimated standard deviation multiplied by the Student t-factor of 1.06 for the original 10 observations and k=1, see Table D.1 on page 40. It will be:

u(x) =1.06 x ±0.074 HRC = ±0.078 HRC

5.2.8 If only a few measurements, *n'*, are made in each test (typically three) and the mean of these n' measurements is quoted to the client as the result, the standard deviation found in a prior experiment should be divided by the square root of the number of measurements for the client, *n'*, to establish the experimental **standard deviation of the arithmetic mean.**

Table 2: Evaluation of standard deviation of the arithmetic mean

Description of operation	Mathematical formulae
Divide the estimated standard deviation of the set of n' measurements by the square root of n', the number of measurements.	$u(x) = \frac{s_{est}}{\sqrt{n'}}$

5.2.9 In the situation that it is not possible to take a large number of repeat readings, the estimated standard deviation from the available readings may be significantly underestimated. If there is a single or clearly dominant source of uncertainty the coverage factor should be based on the student ‘t’ distribution rather than an otherwise assumed normal distribution (see Table D.1 on page 40 and Tables D.5 and D.6). For the case of several sources of uncertainty of roughly equal magnitude, evaluate the residual number of degrees of freedom from the Welch-Satterwaite equation and use the appropriate value of ‘k’ as coverage factor in the evaluation of the expanded uncertainty (see UKAS publication M 3003, Appendix B).

5.2.10 These calculations may be carried out in a systematic way by using a pre-prepared form, which by being completed leads an operator through the necessary steps, by the use of an electronic calculator which has statistical functions, or by means of a computer programme. In all cases the calculation of the original observation and the estimated standard deviation or the standard deviation of the arithmetic mean should be recorded in enough detail for the calculation to be easily checked, at the time or at a later date.

5.3 'Type B' method of evaluation

For Type B estimations the pool of information may include:

- data provided in calibration certificates;
- manufacturer's specifications;
- uncertainties assigned to reference data taken from handbooks;
- previous measurement data;
- experience with, or general knowledge of, the behaviour and properties of relevant materials and instruments; estimations made under this heading are on the basis of considered professional judgements by suitably qualified and experienced personnel and are quite common in many fields of testing.

5.4 Standard Deviation, *s(x)*

5.4.1 The first step in determining uncertainty contributions by the Type B method of evaluation is to estimate the equivalent standard deviation of each contributory component , from whatever source it arises. Such contributions can be derived from a complete statement of uncertainty relating to a previously measured quantity, or from some measure of the uncertainty plus a knowledge, or an inference, of its probability distribution.

5.4.2 A calibration certificate for an instrument used in the test measurement will give the calibrated value plus the uncertainty of measurement together with either its confidence level or the coverage factor used. For some broad range instruments it may be necessary to calculate the uncertainty for the particular reading or instrument range. Unless stated otherwise, assume the uncertainty is a normal distribution and that a quoted 95% confidence level is equivalent to a coverage factor of k = 2, or that a 99.7% or higher confidence level is equivalent to a coverage factor of k = 3. If, exceptionally, no coverage factor is stated, it can only be assumed that the coverage factor used was k = 2. The standard uncertainty from this source is given by the stated, or calculated, uncertainty divided by the coverage factor.

Example; *Establishment of standard uncertainty from a calibration certificate*

The calibration certificate for an instrument used in a test states the measurement uncertainty over its range of calibration as $\pm$0.1% at a 95% confidence level.

The latter can be assumed to be equivalent to the uncertainty being stated with a coverage factor of k = 2.

The contribution of standard uncertainty from the instrument calibration throughout its calibrated range is:

$$u(x) = \pm (0.1 / 2)\% = \pm 0.05\% \text{ of the reading.}$$

5.4.3 Manufacturer's instrument specifications often quote figures without qualification. It can usually be assumed that these are limits within which the instrument was set up or checked, and that a rectangular distribution applies: that is all values within the stated bounds are equally likely. Many other sources of uncertainty where there is no knowledge of the underlying probability distribution, can also be dealt with as a rectangular distribution by assigning reasonable bounds beyond which it is improbable that the contribution to the uncertainty lies. The effects of influence quantities, such as temperature, or the uncertainty due to the digitisation of a measurement (for example, the reading of a Digital Volt Meter), can often be dealt with in this way. The "semi-range" is half of the estimated total range of possible values. The equivalent standard deviation for a rectangular probability distribution is given by the semi-range of the rectangular distribution divided by the square root of 3.

Example; ***Establishment of standard uncertainty from manufacturer's specification***

The manufacturer's specification for an instrument states an accuracy of $\pm$1%.

It can be assumed that this is a statement of the limits of the instrument error and that all values of the error within this band are equally probable (rectangular distribution).

The contribution of standard uncertainty is:

$$u(x) = \pm (1/\sqrt{3})\% = 0.58\% \text{ of the reading.}$$

(Although the final 8 is not significant, it is good practice to retain it as a guard figure rather than round up the uncertainty to $\pm$ 0.6%.)

5.4.4 The exceptional case of asymmetric uncertainty, (for example, operator error in using a vernier micrometer, which is more likely to be positive than negative), can be dealt with by using the mean of the upper and lower bounds as the measured value in the calculation of the result, and a symmetric semi-range of half of the difference.

5.4.5 Uncertainty distributions other than the 'Normal' and 'rectangular' are possible, but unusual. For example, in high frequency electrical measurements, impedance mismatch gives rise to a "U-shaped" uncertainty distribution (uncertainties near the bounds more likely than intermediate values) and in such cases the semi-range should be divided by $\sqrt{2}$ to give the standard deviation. A triangular probability distribution has the divider $\sqrt{6}$.

Example; ***Establishment of standard uncertainty of electrical meter***

An electrical meter is certified to have an uncertainty of 1.0% full scale deflection on its 10 volt range at a confidence level of 95%.

The contribution of standard uncertainty from the calibration of the meter when used on this range, not proportional to the reading, is:

$$u(x) = \pm (10 \times 1.0 \times 0.01 / 2) \text{ volts} = \pm 0.05 \text{ volts}$$

5.5 Standard Uncertainty, *u(x)*

5.5.1 The standard uncertainty is defined as 'one standard deviation'. However, to minimise the possibility of mistakes at a later stage of the evaluation, it is sometimes necessary to multiply values of *Type A* estimated standard deviations and other *Type B* standard deviations by the appropriate sensitivity coefficient to bring them to the same units as the measurand or to take account of other factors in the functional relationship between the input quantities and the output quantity.

5.5.2 When the result of a test is proportional to, or inversely proportional to, a measured quantity, the standard uncertainty of the contribution will be equal to its standard deviation, weighted by a sensitivity coefficient as indicated above where necessary. Consider the example of the measurement of a current which is proportional to a voltage reading, and the measurement of velocity which is inversely proportional to the time of flight between markers.

If Current = Voltage/Resistance, then u(Current) = constant . s(Voltage)

if Velocity = Distance/Time, then u(Velocity) = constant . s(Time)

Influence quantities, such as the effects of temperature, also usually fall into this category.

5.5.3 When the result of a test is proportional to the measured quantity raised to some power, the standard uncertainty of the contribution is the standard deviation of the quantity multiplied by that power. For example, the cross sectional area of a specimen is proportional to the square of its measured diameter, and the standard uncertainty of its area is twice the standard deviation of its diameter. The tensile stress per unit area of a specimen will be inversely proportional to the square of the diameter of the specimen, and again the standard uncertainty will be twice the standard deviation of the measured diameter.

If Area = constant . (Diameter)^2, then s(Area) = constant . 2 . s(Diameter)

5.5.4 In some tests there can be a more complex dependency of the result upon a measured quantity. This can be dealt with by one of two methods. If the functional dependence of the result upon the measured quantity is known, the function can be differentiated to give the ratio of the incremental change of test result corresponding to the incremental change of measured quantity. The standard uncertainty is found by multiplying the standard deviation by the differential factor.

$$\text{If } y = f(a.x)\text{, then } s(y) = \delta f/\delta\,(x)\ .\ a\ .\ s(x)$$

5.5.5 If this is not possible or practical, the result of the test can be calculated for the measured value plus a small increment, equal to its standard deviation, and the measured value minus an equal small increment. Halving the difference between the two results gives the standard uncertainty. Sufficient significant figures must be maintained in the calculation to ensure adequate accuracy.

5.5.6 Some components may be interdependent and could, for example, cancel each other out or could reinforce each other. In certain circumstances such components may be added arithmetically to give a net value. (More rigorous mathematical methods are available for such “correlated” components, and may need to be applied in specific cases, but for many purposes arithmetic addition gives an acceptable estimate for such components). Care must be taken when components tend to cancel each other out, since a small variation in one component may lead to a substantially larger difference. For example, the measured dimension of a specimen using an instrument of similar materials will be sensitive to a lack of temperature stability.

Table 3: Evaluation of standard uncertainty

Description of operation	Mathematical formula
The standard uncertainty is equal to one standard deviation, multiplied by the appropriate sensitivity coefficient (refer to examples)	$u(x_i) = c_i \cdot s_{est}(x_i)$

5.6 Combined standard uncertainty, $u_c(x)$

5.6.1 The component uncertainties need to be combined to produce an overall uncertainty for the measurement. While the choice of method of combination may need to be specially considered in certain cases, the general approach should be to take the square root of the sum of the squares of the component standard uncertainties (known as the root sum square (RSS) method). This operation is simplified if all contributing standard uncertainties have been brought to the same units as outlined above.

Table 4: Evaluation of combined standard uncertainty

Description of operation	Mathematical formula
Take the square root of the sum of the squares of the component standard uncertainties.	$u_c(y) = [u(x_1)^2 + u(x_2)^2 + \ldots]^{½}$

5.6.2 The adoption of the RSS method rather than arithmetic summation takes account of the probability that, in one test, not all the components of uncertainty will have the same sign.

5.7 Expanded uncertainty, U

5.7.1 In most instances of reporting the measurement result of a test to a third party it is necessary to quote an expanded uncertainty at a specified confidence level, the interpretation being that the true value of the measurand lies within the confidence interval given by the stated uncertainty and centred on the reported value with the stated level of confidence. The combined standard uncertainty therefore needs to be multiplied by an appropriate coverage factor. This must reflect the confidence level required and, in the majority of cases, the probability distribution of the combined standard uncertainty can be assumed to be normal, and that a value of k = 2 for the coverage factor defines an interval having a confidence level of approximately 95%. For more critical applications, a value for the coverage factor of 3 defines an interval having a level of confidence of approximately 99.7%.

Table 5: Evaluation of expanded uncertainty

Description of operation	Mathematical Formula
Expanded uncertainty is equal to the combined standard uncertainty multiplied by the coverage factor.	$U = k \cdot u_c(y)$

5.7.2 Exceptions to these cases would need to be dealt with on an individual basis and would be characterised by one or both of the following:

- the absence of a significant number of component uncertainties having well-behaved probability distributions, such as normal and rectangular distributions;
- the domination of the combined standard uncertainty by one large component.

5.7.3 The estimation of the uncertainty of a calibration or test may, in some cases, be confirmed by the results of a proficiency test, a measurement audit or an inter-laboratory comparison. However, the application of statistics to the results of such exercises does not constitute an adequate determination of uncertainty, and taking part in such a scheme does not remove the need to have documented procedures for the estimation of uncertainties.

5.7.4 When infrequent or unusual tests are performed by a laboratory, it may be necessary for the estimation of the measurement uncertainty to be carried out for each test performed. Often, however, tests are of a routine nature using the same equipment and procedures, and are made under similar environmental conditions. In these circumstances it is usual for the uncertainty to be estimated prior to the tests being undertaken and for the values of the measurement uncertainty so determined to be assumed to apply to all similar tests. If this procedure is followed, the components of the uncertainties must allow for that fact by the inclusion of instrument drift, range of environmental conditions, etc.

5.7.5 Care must also be taken if the measurement results may lie over a range of values - some components within the uncertainty budget may have been assumed to be constant over the range, while others can be shown to be proportional to the measured value. In such circumstances it may be necessary to determine the uncertainty at the expected upper end and lower end of the range, and then interpolate the uncertainty value for a particular case. In complex situations, it may be necessary for the range of expected values to be divided into smaller ranges and the applicable uncertainty determined separately for each range.

6 Summary of the steps in estimating uncertainty

6.1 The following is a short, simplified summary of the general procedure to evaluate uncertainty and is applicable in most circumstances. The identification of sources of uncertainty is the most important part of the process. Quantification of uncertainties in testing normally involves a large element of estimation of Type B uncertainty components. Consequently, it is seldom justifiable to expend undue effort in attempting to be precise in the evaluation of uncertainty for testing.

The steps involved are as follows:

1) list all factors which may influence the measured values;

2) make a preliminary estimate of the values of the uncertainty components and place upper bounds on insignificant contributions;

3) estimate the values that are to be attributed to each significant uncertainty component. Express each component in the same manner (ie in the same units or as a percentage, etc) at the one standard deviation level;

4) convert to standard uncertainties, applying sensitivity coefficients where relevant;

5) consider the component uncertainties and decide which, if any, are interdependent and whether a dominant component exists;

6) add any interdependent component uncertainties arithmetically, ie take account of whether they act in unison or in opposition and thereby derive a net value standard uncertainty;

7) take the standard uncertainties of the independent components and the value(s) of any derived net components and, in the absence of a dominant component, calculate the square root of the sum of their squares to produce a combined standard uncertainty.

8) multiply the combined standard uncertainty by a coverage factor k, selected on the basis of confidence level required, to produce an expanded uncertainty. In the absence of a particular confidence level being specified in the standard or by the client, the coverage factor should normally be $k = 2$, giving a confidence level of approximately 95%.

7 Method of stating results

7.1 General approach

7.1.1 Determine the number of significant figures to which the expanded uncertainty will be stated. Due to the assumptions and estimates normally made in determining the uncertainty, this will rarely if ever exceed two significant figures. Round the expanded uncertainty up to the next value of the least significant figure to be retained. Round the test result to the commensurate number of significant figures, in this case to the nearest least significant digit.

7.1.2 The extent of the information given when reporting the result of a test and its uncertainty are normally related to the requirements of the client, or to the intended use of the result, or both. Even if not required in the report, it is good practice to record the following information either in a separate report or in the records of the test:

a) the methods used to calculate the result and its uncertainty;

b) the list of uncertainty components and documentation to show how they were evaluated, ie the sources of data used in the estimation of the components and a record of any assumptions made;

c) sufficient documentation of the steps and calculations in the data analysis to enable an independent repeat of the calculation, should it be necessary;

d) all corrections and constants used in the analysis and their sources.

7.1.3 Unless otherwise specified by the client or in a standard, the result of the measurement should be reported together with the expanded uncertainty appropriate to the 95% confidence level in the following manner:

Measured value	100.1 (units)
Uncertainty of measurement	$\pm$ 0.1 (units)

The reported expanded uncertainty is based on a standard uncertainty multiplied by a coverage factor of k=2, which provides a confidence level of approximately 95%.

7.2 Special cases

7.2.1 In exceptional cases, where a particular factor or factors can influence the results, but where the magnitude cannot be either measured or reasonably assessed, the statement will need to include reference to that fact, for example:

"The reported expanded uncertainty is based on a standard uncertainty multiplied by a coverage factor of k=2, which provides a confidence level of approximately 95%, but excludes the effects of ......"

7.2.2 When the standard uncertainties from many contributions are combined, it can be shown that the distribution of the combined standard uncertainty tends towards a normal (or gaussian) distribution regardless of the distributions of each of the contributions. As an example, three equal rectangular distributions can be shown to combine to give a good approximation to a normal distribution, and one can assert that, with a coverage factor of k = 2, the expanded uncertainty has a confidence level of 95%. For well-behaved distributions (for example, normal, rectangular and U-shaped distributions), with similar magnitudes, a confidence level appropriate to a normal distribution can be associated with the expanded uncertainty.

7.2.3 The method of estimating uncertainties explained in this document can be applied to, and is valid in, all cases, including the case of a single dominant contribution. The combined standard uncertainty is then found from the standard deviation of the single dominant contribution (if the smaller contributions can be totally ignored), and the expanded uncertainty, by definition, is k times this. However, if the distribution of the dominant contribution does not approximate to a normal distribution, it will not be valid to assume that the expanded uncertainty corresponds to a 95% confidence level (for k = 2). The expanded uncertainty is conservative, but it may also be unrealistically large. For example, a rectangular distribution with semi-range 'a' (all values of y fall between $y-a$ and $y+a$ equally likely) has a standard deviation of $a/\sqrt{3}$, and an expanded uncertainty with k = 2 of $\pm 2a/\sqrt{3} = \pm 1.15a$. This is 15% greater than the known limit of the possible error. Thus the concern when a single dominant contribution of uncertainty is met, is not the calculation of the expanded uncertainty which is always valid, but the effect which the distribution of the uncertainty may have on the interpretation of the results of the test; for example, in deciding whether a specification is or is not met.

7.2.4 The criterion for a dominant contribution of uncertainty is often taken to be when it is more than three times the next largest contribution. This is probably too weak and it is advised that special consideration is given if the largest contribution is more than twice the next largest contribution.

7.2.5 If the dominant contribution is known to be, or can be approximated to, a normal distribution, no further special consideration is necessary. A case in point is a dominant contribution found by a Type A evaluation.

7.2.6 When a single dominant contribution does not approximate to a normal distribution, it is good practice to add a note to the presentation of the results of the test, such as:

> "The reported expanded uncertainty is based on a standard uncertainty multiplied by a coverage factor of k = 2, which provides a minimum confidence level of 95%. The distribution of the uncertainty is approximately [rectangular] [U-shaped] [etc]". This should be taken into account in the interpretation of the results of this test".

7.2.7 In making an assessment of compliance with a specification, the distribution of the uncertainty will have no effect when a “shared risk” is applied, or when the uncertainty negligible and can be ignored. When the specification states that the measured result, extended by the uncertainty at a given confidence level, shall not fall outside defined acceptance limits, then it will be necessary to further estimate the value of the uncertainty to be used in the assessment.

7.2.8 Decide upon the distribution of the dominant component. Most distributions of uncertainty contributions which do not approximate to a normal distribution can usually be assumed to be rectangular. If so required by the specification, estimate the confidence limits at the required confidence level, or in the case of a rectangular distribution use the semi-range as the appropriate uncertainty limit. Use this uncertainty value together with the test result to determine whether the latter does or does not meet the specification.

7.2.9 If a statement of compliance or non-compliance is included with the test results, it is good practice to add a note stating the uncertainty limits which were used in the assessment if these are different from the expanded uncertainty.

7.2.10 Any uncertainty that results from the test sample not being fully representative of the whole should normally be excluded from the evaluation of uncertainty of measurement. However, there may be insufficient information to enable this to be done and, in that case, this must be stated in the report of uncertainty. There may be special cases when it is considered necessary to quote a separate value for the uncertainty due to variation between samples. Such cases must be considered individually in order to decide on the procedure for determining the required value. If the laboratory is responsible for taking samples, the method stated in a standard specification or documented procedure should be followed. When requested by the client, the spread of values in the test due to the sampling should be determined.

8 Conclusions

8.1 This document gives a grounding in measurement uncertainty for all areas of testing. It does not, and cannot, give guidance in specific cases. For example, even in one field of testing the effects which contribute to uncertainty may vary with the equipment used to carry out the test, and their magnitudes will vary with the manner in which the equipment is used. Examples are useful to give understanding of the principles, but they are no substitute for the in-depth consideration of each case which is required.

8.2 The methods outlined here are applicable to the majority of tests. However, a number of assumptions are required to be made for this to be the case. These assumptions must be kept in mind whenever an uncertainty is calculated, and if they cannot be demonstrated to be applicable, recourse to other documents is necessary for resolution of the procedure to be followed.

8.3 The assumptions in this document which permit its simplified procedures to be followed are:

a) the combined uncertainty can be evaluated as the combination of component contributions;

b) all significant contributions have been identified;

c) all contributions from whatever source can be converted with sufficient accuracy into a standard uncertainty;

d) the contributions to the uncertainty are statistically independent, except where allowance for interdependence has been applied;

e) where components of uncertainty are interdependent, the effect can be allowed for by the arithmetic summation of the components which may then be treated as one independent component;

f) the summation by the root sum square, (RSS), method leads to a probability distribution which is close to normal, so that the combined uncertainty may be multiplied by a coverage factor to give an expanded uncertainty with an approximately known confidence level;

g) there is no contribution to the uncertainty which dominates over all others, and which has a probability distribution which cannot be treated by the simplified methods presented here.

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Glossary

This glossary gives explanations for some of the terms and words used within the text. Precise and rigorous definitions for many of them can be found in *The Guide* and in BSI PD 6461, and in the '*International vocabulary of basic and general terms used in metrology*' and other sources listed in the Bibliography. The meanings of some terms have been given in the text when they first occur but are repeated here for convenience of reference.

Arithmetic mean

sum of measured values divided by the number of values

Accuracy

closeness of the agreement between a measurement result and a true value. As the true value is not known, accuracy is a qualitative term only.

Bias of a measuring instrument

a systematic error source, relating to the indication of a measuring instrument.

Calibration

comparison of an instrument against a reference or a standard, to show errors in the values indicated by the instrument.

Combined standard uncertainty

result of the combination of standard uncertainty components.

Confidence level

a number expressing the degree of confidence in a quoted result, e.g. 95%. The probability that the value of the measurand lies within the quoted range of uncertainty.

Correlation

interdependence or relationship between data or between measured quantities.

Coverage factor

numerical factor used to multiply the combined standard uncertainty to give the expanded uncertainty at a specified level of confidence.

Error of measurement

the result of a measurement minus the value of the measurand, (not precisely quantifiable because the true value is unknown within the range of uncertainty).

Estimated standard deviation

the estimate of the standard deviation of the 'population based on a limited sample'.

Expanded uncertainty

an interval about the measurement result that may be expected to encompass a large, specified fraction (eg 95%) of the distribution of values that could be reasonably attributed to the measurand.

Functional relationship
the relationship between the measurand and the input quantities on which it depends.

Measurand
specific quantity subject to measurement.

Measurement error
difference between an indicated value and a presumed true value.

Measurement uncertainty
a measure of the bounds within which a measurand may be presumed to lie with a given probability.

Normal probability distribution
distribution of values in a characteristic pattern of spread, also known as a Gaussian distribution.

Partial derivative
differential with respect to an individual input quantity of the functional relationship between the measurand and the input quantities.

Precision
term relating to 'fineness of discrimination', sometimes incorrectly used to mean 'accuracy'.

Probability distribution
a function giving the probability that the random variable takes at any given value or belongs to a set of values.

Random errors
errors whose effects are observed to vary in a random manner.

Range
difference between the highest and lowest of a set of values.

Rectangular probability distribution
distribution of values with equal likelihood of falling anywhere within a defined range.

Sensitivity coefficient (associated with an input quantity)
the differential change in the output generated by a differential change in an input divided by the change in that input, $\delta f/\delta x_i$.

Sensitivity coefficient (conversion of units)
coefficient used to multiply an input quantity to express it in terms of the output quantity, e.g. temperature coefficient of material under test.

Standard deviation
the positive square root of the variance, giving a measure of the distribution of values in a Normal probability distribution.

Standard deviation of arithmetic mean
the standard deviation of a set of measurements divided by the square root of the number measurements in the set.

Standard uncertainty
the estimated standard deviation, weighted by the sensitivity coefficient.

Student t-factor
a multiplying factor to compensate for underestimation of a standard deviation for a reduced number of measurements.

Systematic error
bias or offset errors, either positive or negative, from the correct value.

True value
the value that would be obtained by a perfect measurement.

Tolerance
the limiting or permitted range of values of a defined quantity. Also used to indicate the acceptable range of values indicated by an instrument when set up and tested by the manufacturer.

Type A evaluation of uncertainty
evaluation of uncertainty of measured values by statistical methods.

Type B evaluation of uncertainty
evaluation of uncertainty of single measured or estimated values by non-statistical methods.

Uncertainty budget
summary of sources of uncertainty and their magnitudes with their summation to give the final expanded uncertainty of measurement.

Uncertainty
a parameter, associated with the result of a measurement, that characterises the dispersion of the values that could reasonably be attributed to the measurand.

Variance
a measure of the dispersion of a set of measurements, giving a measure of the distribution of values in a Normal probability distribution.

Appendix A: Method development and validation

A.1 Method development and validation

A.1.1 The fitness for purpose of specific test methods applied for routine testing is most commonly assessed during method development and through method validation studies. Such studies produce data on overall performance and on individual influence factors, which can be applied to the estimation of uncertainty associated with the results of the method in normal use.

A.1.2 Method validation studies rely on the determination of overall method performance parameters. These are obtained during method development and interlaboratory study or following in-house validation protocols. During method development individual sources of error or uncertainty are typically investigated only when significant compared to the overall precision measures in use. The emphasis is primarily on identifying and removing (rather than correcting for) significant effects. This leads to a situation in which the majority of potentially significant influence factors have been identified, checked for significance compared to overall precision, and shown to be negligible. Under these circumstances, the data available consists primarily of overall performance figures, together with evidence of insignificance of most effects and some measurements of any remaining significant effects.

A.1.3 Validation studies for quantitative testing methods typically determine some or all of the following parameters:

> <u>Precision.</u> The principal precision measures include **repeatability** standard deviation s_r, **reproducibility** standard deviation s_R, and intermediate **precision**, sometimes denoted s_{zi}, with i denoting the number of factors varied. The repeatability s_r indicates the variability observed within a laboratory, over a short time, using a single operator, item of equipment *etc.* s_r may be estimated within a laboratory or by inter-laboratory study. Interlaboratory reproducibility standard deviation s_R for a particular method may only be estimated directly by interlaboratory study; it shows the variability obtained when different laboratories analyse the same sample. Intermediate precision relates to the variation in results observed when one or more factors, such as time, equipment and operator, are varied within a laboratory; different figures are obtained depending on which factors are held constant. Intermediate precision estimates are most commonly determined within laboratories but may also be determined by interlaboratory study. The observed precision of a testing procedure is an essential component of overall uncertainty, whether determined by combination of individual variances or by study of the complete method in operation.
>
> <u>Bias.</u> The bias of a testing method is usually determined by study of relevant reference materials. The determination of overall bias with respect to appropriate reference values is important in establishing traceability to recognised measurement standards. Bias should be shown to be negligible or corrected for, but in either case the uncertainty associated with the determination of the bias remains an essential component of overall uncertainty.

Linearity. Linearity is an important property of methods used to make measurements at a range of values. The linearity of the response to standards and to realistic samples may be determined. Linearity is not generally quantified, but is checked for by inspection or using significance tests for non-linearity. Significant non-linearity is usually corrected for by use of non-linear calibration functions or eliminated by choice of more restricted operating range. Any remaining deviations from linearity are normally sufficiently accounted for by overall precision estimates covering several values, or within any uncertainties associated with calibration.

Robustness or ruggedness. Many method development or validation protocols require that sensitivity to particular parameters be investigated directly. This is usually done by a preliminary 'ruggedness test', in which the effect of one or more parameter changes is observed. If significant (compared to the precision of the ruggedness test) a more detailed study is carried out to measure the size of the effect, and a permitted operating interval chosen accordingly. Ruggedness test data can therefore provide information on the effect of important parameters.

A.2 Conduct of experimental studies of method performance

A.2.1 The main principles as they affect the relevance of the results of a study to uncertainty estimation are considered below.

Representativeness is essential. Studies should, as far as possible, be conducted to provide a realistic survey of the number and range of effects operating during normal use of the method, as well as covering the range of values and sample types within the scope of the method. Where a factor has been representatively varied during the course of a precision experiment, for example, the effects of that factor appear directly in the observed variance and need no additional study unless further method optimisation is desirable.

In this context, *representative variation* means that an influence parameter must take a distribution of values appropriate to the uncertainty in the parameter in question. For continuous parameters, this may be a permitted range or stated uncertainty; for discontinuous factors such as type of sample, this range corresponds to the variety of types permitted or encountered in normal use of the method. Note that representativeness extends not only to the range of values, but to their distribution.

Where factors are known or suspected to interact, it is important to ensure that the effect of interaction is accounted for. This may be achieved either by ensuring random selection from different levels of interacting parameters, or by careful systematic design to obtain both variance and covariance information.

In carrying out studies of overall bias, it is important that the reference materials and values are relevant to the materials under routine test.

A.3 Quantifying Uncertainty

A.3.1 In order to evaluate the uncertainty it is first necessary to prepare a comprehensive list of relevant sources of uncertainty. In forming the required list of uncertainty sources it is usually convenient to start with the basic expression used to calculate the measurand from intermediate values. All the parameters in this expression may have an uncertainty associated with their value and are therefore potential uncertainty sources. In addition there may be other parameters that do not appear explicitly in the expression used to calculate the value of the measurand, but which nevertheless affect the measurement results, e.g. temperature. These are also potential sources of uncertainty that should be included.

A.3.2 The use of a 'cause and effect diagram' is a very convenient way of listing the uncertainty sources, showing how they relate to each other and indicating their influence on the uncertainty of the result. It also helps to avoid double counting of sources.

A.3.3 Most of these sources of uncertainty are likely to have been investigated during validation studies or from other experimental work that has been carried out to check the performance of the method. However, data may not be available to evaluate the uncertainty from all of the sources and it may be necessary to carry out further work to evaluate the uncertainty arising from these.

The sources that may need particular consideration are:

Sampling. Collaborative studies rarely include a sampling step. If the method used in-house involves sub-sampling, or the measurand is estimating a bulk property from a small sample, then the effects of sampling should be investigated and their effects included.

Pre-treatment. It may be necessary to investigate and add the effects of the particular pre-treatment procedures applied in-house.

Method bias. Method bias is often examined prior to or during interlaboratory study, where possible by comparison with reference methods or materials. Where the bias itself, the uncertainty in the reference values used, and the precision associated with the bias check, are all small compared to s_R, no additional allowance need be made for bias uncertainty. Otherwise, it will be necessary to make additional allowances.

Variation in conditions. Laboratories participating in a study may tend towards the mean of allowed ranges of experimental conditions, resulting in an underestimate of the range of results possible within the method definition. Where such effects have been investigated and shown to be insignificant across their full permitted range, no further allowance is required.

Changes in sample type. The uncertainty arising from samples with properties outside the range covered by the study will need to be considered.

A.3.4 For sources of uncertainty not adequately covered by existing data, either seek additional information from the literature or standing data (certificates, equipment specifications *etc.*), or plan experiments to obtain the required additional data. Additional experiments may take the form of specific studies of a single contribution to uncertainty, or the usual method performance studies conducted to ensure representative variation of important factors.

A.3.5 It is important to recognise that not all of the components will make a significant contribution to the combined uncertainty; indeed, in practice it is likely that only a small number will. Unless there is a large number of them, components that are less than one third of the largest need not be evaluated in detail. A preliminary estimate of the contribution of each component or combination of components to the uncertainty should be made and those that are not significant eliminated.

A.3.6 In order to use the results of prior studies of the method to evaluate the uncertainty, it is necessary to demonstrate the validity of applying prior study results. Typically, this will need to demonstrate that a comparable precision to that obtained previously can be achieved and also that the use of the bias data obtained previously is justified, typically through determination of bias on relevant reference materials (see, for example, ISO Guide 33), by satisfactory performance on relevant proficiency schemes or other laboratory intercomparisons. Continued performance within statistical control as shown by regular QC sample results and the implementation of effective analytical quality assurance procedures. Where the conditions above are met, and the method is operated within its scope and field of application, it is normally acceptable to apply the data from prior studies (including validation studies) directly to uncertainty estimates in the laboratory in question.

A.3.7 For methods operating within their defined scope, when the reconciliation stage shows that all the identified sources have been included in the validation study or when the contributions from any remaining sources have been shown to be negligible, then the reproducibility standard deviation s_R, may be used as the combined standard uncertainty.

A.3 8 If there are any significant sources of uncertainty that are not included in the validation study then their contribution needs to be evaluated separately and combined with the reproducibility standard deviation, s_R , to obtain the combined standard uncertainty.

Glossary:

Precision - the closeness of agreement between independent test results obtained under stipulated conditions.

Repeatability - precision under conditions where independent test results are obtained with the same method on identical test items in the same laboratory by the same operator using the same equipment within short intervals of time.

Reproducibility - precision under conditions where test results are obtained with the same method on identical test items in different laboratories with different operators using different equipment.

Appendix B: Assessment of Compliance with Specification

B.1 When a test is carried out to a stated specification and the specification requires a statement of compliance the test report should contain a statement stating whether or not the results indicate compliance with the specification. There are a number of possible cases where the uncertainty of measurement has a bearing on the compliance statement which will be examined.

B.2 In the simplest case the specification requires that the measured result extended by the uncertainty, at a given level of confidence, shall not fall outside a defined limit. In these cases assessment for compliance is straightforward.

B.3 Unfortunately clear statements of the compliance conditions are rare, even in written standards. More often the specification requiring a compliance statement makes no reference to taking into account the uncertainty of related measurements. In such cases it may be appropriate for the user/customer to make the judgement of compliance based on whether the result is within the specified limits with no account taken of the uncertainty of measurement. This approach is often referred to as 'shared risk' since the end user takes some of the risk that the product may not meet the specification.

B.4 This approach may be adopted if the magnitude of the measurement uncertainty is considered to be acceptably small and that it will be considered when necessary. However, this requires that the measuring laboratory should always be in the position to determine the magnitude of the uncertainty if requested to do so.

> An example of this approach in to be found in the British Standard specification for the manufacture of Gauge Blocks, BS 4311:1993. The published tolerance on the length dimensions to which they should be manufactured takes no account of the measurement uncertainty in the decision on compliance with the specification. In practice the uncertainty of measurement may be up to one fifth, 20%, of the length tolerance, reducing the manufacturing tolerance to only four fifths of the specified tolerance value if the manufacturer is to ensure that his product is in compliance with the BS specification. In the case of used gauge blocks the uncertainty of measurement may be up to 60% of the length tolerance.

B.5 In the absence of any specified criteria, test specifications or codes of practice, the following approach is recommended:

1). if the tolerance limits are not exceeded, top or bottom limits, by the measured result extended by the expanded uncertainty interval, at the 95% confidence level, then compliance with the specification is achieved, (example 1 in the figure)

2) When the upper tolerance level is exceeded by the measured result reduced by half of the expanded uncertainty interval, then non-compliance with the specification results, (example 2). Similarly, if the lower tolerance level is not reached by the measured result plus half of the expanded uncertainty interval, there is non-compliance with the specification, (example 3).

3) If the measured value falls close enough to either top or bottom tolerance limit that the expanded uncertainty interval overlaps the limit then It is not possible to confirm either compliance or non-compliance at the stated confidence level, (examples 4 and 5).

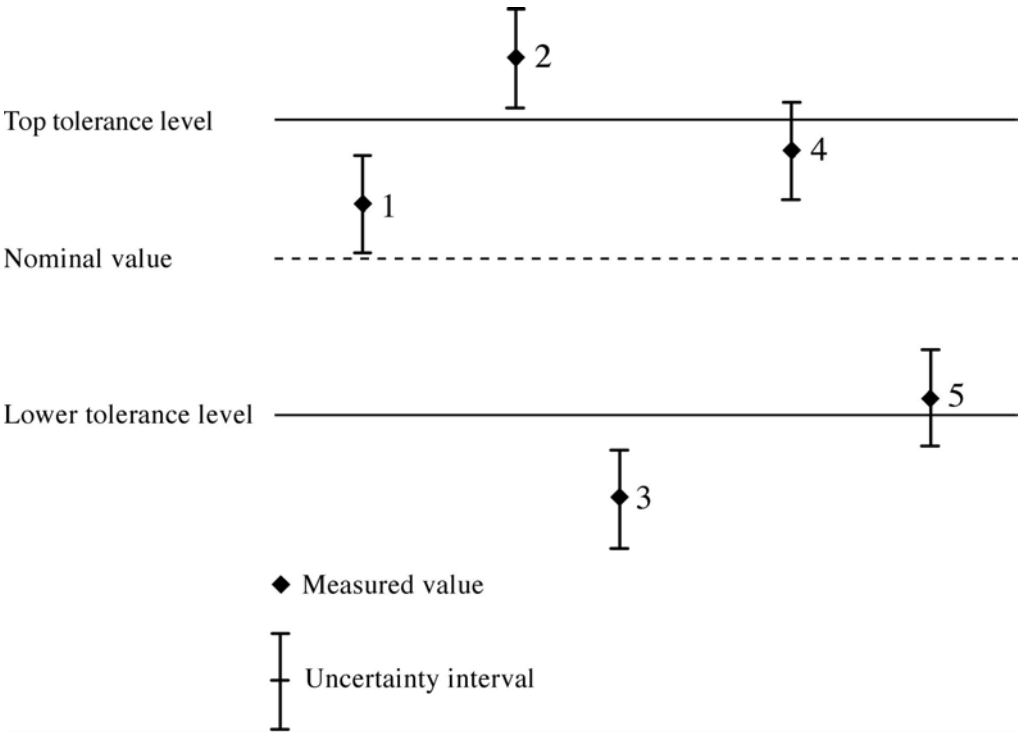


Figure B.1, Assessing compliance when the measured result includes the influence of measurement uncertainty.

Appendix C: Best Measurement Capability

"The smallest uncertainty of measurement that a laboratory can achieve within its scope of accreditation, when performing more or less routine calibrations/tests of nearly ideal measurement standards intended to define, realise, or reproduce a unit of quantity or when performing more or less routine calibrations/tests of nearly ideal measuring instruments designed for the measurement of a quantity".

(EA-4/02 Expression of the Uncertainty of Measurement in Calibration)

C.1 Best measurement capability is one of the parameters that is used to define the capabilities of an accredited laboratory and is normally stated in its accreditation schedule. It is one of the essential pieces of information to be quoted in directories of accredited laboratories and is used by potential customers of those laboratories to judge the suitability of a laboratory to carry out particular measurements.

C.2 The phrase "more or less routine calibration/test" means that the laboratory shall be able to achieve the stated capability in the normal work that it performs under its accreditation. However, there can be circumstances where the laboratory is able to do better, maybe as a result of extensive investigations and additional precautions. These cases are not covered by the definition of best measurement capability given above, unless it is the policy of the laboratory to perform such extensive investigations in all testwork, in which case these then become the "more or less routine" measurements by the laboratory.

C.3 Inclusion of the qualifier "nearly ideal measurement standards" in the definition means that the best measurement capability should not be dependant on the characteristics of the device to be investigated. This implies that there should be only the minimum contribution to the uncertainty of measurement attributable to the physical effects that can be ascribed to imperfections of the device under investigation. It does require that such a device is actually available to the laboratory. If it is established that even the most "ideal" available device contributes significantly to the uncertainty of measurement, this contribution must be included in the evaluation of the best measurement capability and, if necessary, a statement should be included that the statement of best measurement capability refers to measurements made with that type of device.

C.4 The definition of best measurement capability implies that within its accredited activities a laboratory cannot claim a smaller uncertainty of measurement than the best measurement capability. In its normal activities the laboratory shall be required to state a larger uncertainty than that corresponding to the best measurement capability whenever it is established that the actual measurement process adds significantly to the uncertainty of measurement. Typically the test method used will give a contribution and so the actual uncertainty of measurement can never be smaller than the best measurement capability.

C.5 Assessment of the best measurement capability is the task of the accreditation body. The estimation of the uncertainty of measurement that defines the best measurement capability shall follow the currently accepted procedures and the best measurement capability shall be stated at the same level of probability as required for test reports and certificates, i.e. in the form of an expanded uncertainty of measurement with a coverage factor corresponding to 95% probability. All components contributing significantly to the uncertainty of a measurement shall be taken into account when evaluating the best measurement capability. None are to be omitted on the basis of not occurring with an ideal device.

C.6 The evaluation of contributions to the uncertainty that are known to vary with time or with some other physical quantity should be based on limits of possible variations as would occur under normal working conditions. For instance, if a used working standard is known to drift with time, the contribution caused by the drift between calibrations of the standard must be estimated and included when estimating the uncertainty contribution of the working standard.

C.7 As an example of the evaluation of the best measurement capability, consider the evaluation of the measurement uncertainty of a polarimeter using quartz control plates. The uncertainty sources contributing to the expanded uncertainty are listed in the Table C.1. These include such items as the uncertainty in the calibration of the standard quart control plates by a National Standards Laboratory, the correction of their values as a function of measured temperature and their use to check instrument linearity and the reading of the instrument display. Table C.1 presents the estimated expanded uncertainty of measurement achieved at for a typical measurement with the polarimeter. The value of the expanded uncertainty achieved in normal circumstances is $\pm$0.043 degrees rotation.

C.8 Table C.2 presents an estimate of the measurement uncertainty that would be achieved using the definition of best measurement capability. The estimate is in terms of a "more or less routine measurement" but using the most sensitive polarimeter available to the company and a well controlled operating environment, etc. The value of the expanded uncertainty achieved under the conditions of best measurement capability would be $\pm$0.011 degrees rotation and it is this value that should be presented in the company's scope of accreditation.

Table C.1: Best Measurement Capability, expanded uncertainty under normal conditions

Source of Uncertainty	Value	Divisor	Sensitivity Coeff.	Standard Uncertainty
Type A Observed variation in Readings	+0.007°	1	1	+0.007°
Type B Calibration of Standard QCPs	±0.001°	2	1	±0.0005°
Drift between calibrations	±0.001°	2	1	±0.0005°
Temperature compensation factor				
Thermometer calibration	±0.13°C	√3		
Temp compensation factor			0.000144	
Evaluated factor for 33.5° QCP				±0.00035°
Resolution of polarimeter	±0.025°	√3	1	±0.0145°
Linearity test	±0.025°	√3	1	±0.0145°
Combined uncertainty	±0.021°			
Expanded uncertainty	±0.043°			

Table C.2: Best Measurement Capability, estimated best achievable expanded uncertainty

Source of Uncertainty	Value	Divisor	Sensitivity Coeff	Standard Uncertainty
Type A Observed variation in Readings	+0.0035°	1	1	+0.0035°
Type B Calibration of Standard QCPs	±0.001°	2	1	±0.0005°
Drift between calibrations	±0.001°	2	1	±0.0005°
Temperature compensation factor				
Thermometer calibration	±0.10°C	√3		
Temp compensation factor			0.000144	
Evaluated factor for 33.5° QCP				±0.00029°
Resolution of polarimeter	±0.005°	√3		±0.0029°
Linearity test	±0.005°	√3		±0.0029°
Combined uncertainty	±0.0055°			
Expanded uncertainty	±0.011°			

Appendix D: Examples

Example D.1; *Establishment of Standard uncertainty from calibration certificate*

The calibration certificate for an instrument used in a test states the measurement uncertainty over its range of calibration as $\pm 0.1\%$ at a 95% confidence level.

The latter can be assumed to be equivalent to the uncertainty being stated with a coverage factor of $k = 2$.

The contribution of standard uncertainty from the instrument calibration throughout its calibrated range is:

$$u(x) = \pm (0.1 / 2)\% = \pm 0.05\% \text{ of the reading.}$$

Example D.2; ***Establishment of Standard uncertainty from manufacturer's specification***

The manufacturer's specification for an instrument states an accuracy of $\pm 1\%$.

It can be assumed that this is a statement of the limits of the instrument error and that all values of the error within this band are equally probable (rectangular distribution).

The contribution of standard uncertainty is:

$$u(x) = \pm (1/\sqrt{3})\% = 0.58\% \text{ of the reading.}$$

> *Note: Although the final 8 is not significant, it is good practice to retain it as a guard figure for the next stage in the calculations rather than round up the uncertainty to $\pm$ 0.6% at this stage.*

Example D.3; *Establishment of Standard uncertainty of electrical meter*

An electrical meter is certified to have an uncertainty of 1.0% full scale deflection on its 10 volt range at a confidence level of 95%.

The contribution of standard uncertainty from the calibration of the meter when used on this range, not proportional to the reading, is:

$$u(x) = \pm (10 \times 1.0 \times 0.01 / 2) \text{ volts} = \pm 0.05 \text{ volts}$$

Example D.4; *Establishment of standard uncertainty from a standard specification*

A vernier caliper is certified to comply with BS887:1982 which states that the deviation of the reading of the instrument up to 300 mm shall not exceed 0.02 mm. This can be assumed to be equivalent to a rectangular distribution of bounds $\pm$0.02 mm.

The length of a specimen measured with the vernier caliper is found to be 215.76 mm and the contribution of standard uncertainty from the calibration of the instrument is:

$$\pm (0.02 / \sqrt{3})\ \text{mm} = \pm 0.012\ \text{mm, or } \pm 0.0054\% \text{ of the reading.}$$

Note: other uncertainty sources, such as temperature, coefficient of expansion, must also be included in the evaluation of the expanded uncertainty.

Example D.5; *Establishing the expanded uncertainty in a Charpy impact strength test.*

A number of similar specimens of the same plastic material are subjected to Charpy impact strength tests using Method 359 of BS 2782: Part 3: 1993. The first five specimens all failed by buckling. After correcting for friction of the pendulum, calculation gave the following values of the impact strength:

89.9 85.6 90.4 82.9 84.5 kJ/m^2

The standard deviation of the impact strength is calculated manually in the following table:

	Value kJ/m^2	Residual kJ/m^2	Residual^2 (kJ/m^2)^2
	89.9	+3.84	14.75
	85.6	-0.46	0.21
	90.4	+4.34	18.84
	82.9	-3.16	9.99
	81.5	-4.56	20.79
Sum = 430.3		Sum = 64.58	
Mean = 86.06		Standard deviation = s(x) = 4.02	

The standard uncertainty is $\pm$4.02 kJ/m^2, or $\pm$4.7% of the reading. BS 2782 states that five measurements are sufficient if the standard deviation is less than 5% of the mean reading.

Other contributions to the combined uncertainty have been considered and are determined to be each less than $\pm$0.5%. They may be regarded as insignificant, leaving only the uncertainty arising from the Charpy impact tests. For 5 measurements and a coverage factor of k = 2, Table D.1 gives a value for the student t-factor of 1.44. The expanded uncertainty is thus:

$$U = \pm (2 \times 1.44 \times 4.02//5) = \pm 5.1\ \text{kJ/m}^2$$

and the rounded-up measured value, at 95% confidence, is 86 $\pm$ 6 kJ/m^2.

Example D.6; ***Establishment of expanded uncertainty from a single measurement***

This example establishes the total expanded uncertainty derived from a single measurement from a previously calibrated machine and includes further contributing Type B uncertainties

During preliminary evaluation of the repeatability of operation of a Rockwell hardness operating system, 10 indentations were made on a Rockwell hardness test block with the following readings:

45.4 45.5 45.4 45.3 45.5 45.3 45.3 45.4 45.4 45.4 HRC

The related statistical parameters are: Mean value 45.39 HRC

Standard deviation ±0.074 HRC

For a single reading made with the same system on a subsequent occasion, the contribution to the standard uncertainty, including the Student t-factor for 10 observations and k=1 as given in Table D.1 as 1.06, will be:

$$u(x) = 1.06 \times \pm 0.074 \text{ HRC} = \pm 0.078 \text{ HRC}$$

Other sources of Type B uncertainty were identified the follows:

- The indenter had been verified to 0.3 HRC and the depth measuring device to 0.1 HRC.
- The standardizing machine, against which the hardness machine had been indirectly verified, had been verified to 0.5 HRC (the limits set by BS 891).
- The uncertainty associated with the total force, which was better than ±0.1%, gave a negligible contribution to the total uncertainty (<0.01 HRC), as did the uncertainty associated with the diameter of the indenter balls.

The functional relationship for the measurement was identified as a linear combination. The total expanded uncertainty derived from a single measurement on the machine and other uncertainty sources, using a coverage factor of k = 2 for 95% probability, is evaluated in the following table:

Source of Uncertainty	Standard Deviation	Divisor	Sensitivity Coeff.	Standard Uncertainty
Single reading of hardness	±0.074	1	1.06	±0.078
Indenter verification	±0.03	√3	1	±0.017
Indenter measurement	±0.10	√3	1	±0.058
Standardising machine verification	±0.50	√3	1	±0.289
Combined uncertainty		±0.305		
Expanded uncertainty, k=2		±0.61		

Example D.7; *Measurement of volume of liquid passing through a flowmeter*

The volume of liquid passed through an oil flowmeter is determined from measurements of the density of the fluid and the mass of fluid collected.

Information available from the recorded notes of a measurement run give the following:

- The density of the fluid is found by weighing a known volume at a measured temperature. The uncertainty of this density measurement, including the effect of temperature, was ±0.006% (with a 95% confidence probability).
- The uncertainty of the density due to temperature measurement in the flow line was ±0.004% (with a 95% confidence probability).
- The contributions of uncertainty from the density measurements are not independent but are correlated. These two contributions are therefore summed arithmetically to give an uncertainty of ±0.01% at a 95% confidence level.
- The uncertainty in the measurement of the mass was ±0.013% (with a 95% confidence probability).
- Other contributions were shown to be negligible.

The functional relationship for the measurements indicates correlation of the density measurements. Summation of the standard uncertainties takes account of this. The expanded uncertainty of the volume of liquid is evaluated in the following table. All values of standard deviation and standard uncertainty are in percentage units.

Source of Uncertainty	Value	Divisor	Sensitivity Coeff.	Standard Uncertainty
Correlated contributions				
Density measurement (mass)	±0.006	2	1	±0.003
Density measurement (flow)	±0.004	2	1	±0.002
Arithmetic sum	±0.01			±0.005
Un-correlated contributions				
Mass measurement	±0.013	2	1	±0.0065
Other contributions, negligible	---			---
Combined uncertainty		±0.008		
Expanded uncertainty, k=2		±0.016		

Example D.8; *Measurement of Lens Focal Length*

The measurement is of the focal length of a test lens, such as a camera lens, using a collimated target as source and a nodal slide test bench to find the distance from the rear nodal plane of the lens to its focal plane. This distance is defined as the lens equivalent focal length.

Measurement Procedure:

1) Place lens in the holder on Lens Test Bench, set up target in focal plane of collimator and illuminate as required. Use 'focus micrometer' control to focus target image on image plane graticule. Make ten (10) settings of 'best focus', recording the micrometer reading each time.

2) Calculate the mean value and the estimate of the standard deviation for the set of ten readings. Reset the 'focus micrometer' to the mean value.

3) Make five (5) settings of the 'nodal pivot slide', noting the pivot setting micrometer each time. Calculate the mean value and the estimate of the standard deviation for the set of five readings. Reset the pivot setting micrometer to the mean value.

4) Take a single reading of the focal length of the lens under test from the focal length scale attached to the lens test bench, using the vernier to estimate to the nearest 0.001 inch.

The functional relationship indicates a linear combination of measurements.

Source of Uncertainty	Value	Divisor	Coeff.	Standard Uncertainty
Type A Focus setting, 10 readings Estimated Standard deviation Standard deviation of the Mean	±0.0004	√10	1	±0.00012
Nodal pivot setting, 5 readings Estimated Standard deviation Standard deviation of the Mean	±0.0011	√5	1	±0.00049
Type B Error in reading focal length from scale	±0.0010	√3	1	±0.00058
Focal length scale zero positioning error	±0.0010	√3	1	±0.00058
Temperature compensation for scale Temperature range Expansion coefficient (steel) Evaluated factor	±2.0 °C	√3	0.000012/°C	±0.000014
Focusing error of microscope on graticule	±0.00010	√3	1	±0.000058
Collimation error	±0.00001	√3	1	±0.000006
Combined uncertainty		±0.00096		
Expanded Uncertainty, (k=2)		±0.002		

Example D.9; *Measurement uncertainty of an Aspirated Hygrometer*

An aspirated hygrometer is a frequently used instrument in standards laboratories to determine the relative humidity of laboratory air. This value may then be used in the evaluation of the refractive index of the air and the correction of wavelength of light used in interferometers etc.

Measurement Procedure:

1) Dampen the wick attached to the wet bulb thermometer, set fan of the hygrometer running, allow readings of the wet and dry bulb thermometers to stabilise.
2) Take five (5) readings of the temperature indicated by the dry bulb. Calculate the <u>mean value</u> and the <u>estimate of the standard deviation</u>.
3) Take ten (10) readings of the temperature indicated by the wet bulb. Calculate the <u>mean value</u> and the <u>estimate of the standard deviation</u>.
4) Apply corrections to the mean temperatures as indicated on calibration certificates.
5) Calculate the depression of the wet bulb temperature.

The functional relationship indicates a linear combination of measurements.

Source of Uncertainty	Value	Divisor	Coeff.	Standard Uncertainty
Type A				
Thermometer 1, Dry Bulb, 5 readings				
Estimated Standard deviation	±0.033 °C			
Standard deviation of the Mean		√5	1	±0.015 °C
Thermometer 2, Wet Bulb, 10 readings				
Estimated Standard deviation	±0.033 °C			
Standard deviation of the Mean		√10	1	±0.010 °C
Type B				
Uncertainties in thermometer calibrations (2 thermometers)	±0.010 °C	2	1	±0.005 °C
	±0.010 °C	2	1	±0.005 °C
Estimated influence of water purity	±0.007 °C	√3	1	±0.005 °C
Combined uncertainty	±0.022 °C			
Expanded uncertainty, k = 2.4	±0.053 °C			

Notes: 1) The quoted result is in terms of the uncertainty in depression of the wet bulb temperature. The uncertainty in the measured relative humidity value will depend on the dry bulb temperature and the depression of the wet bulb temperature and should be determined from the tables supplied with the hygrometer for the determination of relative humidity.

2) The increased value of the coverage factor to 2.4 results from an assessment of the residual number of degrees of freedom arising from the small number of measurements of the two largest standard uncertainties. See reference 5, Bibliography.

Example D.10; ***On-site calibration of a Polarimeter***

The polarimeter is tested for correct operation within its performance specification after an annual service on-site at the customer's laboratory. This is achieved by check measurements using calibrated standard quartz control plates that cover its range of operation. The performance specification is in the form of an 'accuracy of ±0.05 degrees optical rotation over a range of ±35 degrees optical rotation'. The uncertainty of measurement is therefore evaluated for the highest value in the range of optical rotation, with linearity assessed at other measured values within the range for which standard quartz control plates are available.

The functional relationship for the measurements indicates linear combination of non-correlated parameters.

Source of Uncertainty	Value	Divisor	Sensitivity Coeff.	Standard Uncertainty
Type A				
Observed variation in Readings Standard deviation	±0.007°	1	1	±0.007°
Type B				
Calibration of Standard QCPs	±0.001°	2	1	±0.0005°
Drift between calibrations	±0.001°	2	1	±0.0005°
Temperature compensation factor				
Thermometer calibration	±0.13°C	√3		
Temp compensation factor			0.000144	
Evaluated factor for 33.5° QCP				±0.00035°
Resolution of polarimeter	±0.025°	√3	1	±0.0145°
Linearity test	±0.025°	√3	1	±0.0145°
Combined uncertainty		±0.0215°		
Expanded uncertainty, k=2		±0.043°		

Table D.1; Correction to be applied to the standard deviation for the number of measurements

	Correction T = t/k		
n	k = 1	k = 2	k = 3
3	1.32	2.27	-
4	1.20	1.66	3.07
5	1.14	1.44	2.21
6	1.11	1.33	1.84
7	1.09	1.26	1.63
8	1.08	1.22	1.51
9	1.07	1.19	1.43
10	1.06	1.16	1.36
11	1.05	1.14	1.32
12	1.05	1.13	1.28
13	1.04	1.12	1.25
14	1.04	1.11	1.23
15	1.04	1.10	1.21
16	1.03	1.09	1.20
17	1.03	1.09	1.18
18	1.03	1.08	1.17
19	1.03	1.08	1.16
20	1.03	1.07	1.15

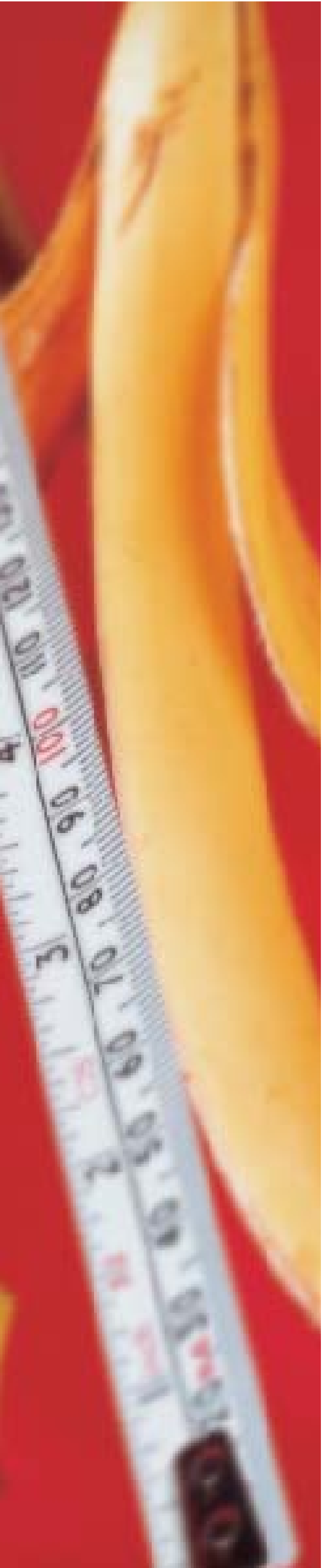
Examples: a) For n=10 measurements and k=1, the correction factor is 1.06

i.e. standard uncertainty = 1.06 x estimated standard deviation

b) For n=5 measurements of the sole source of uncertainty at a 95% confidence level, k=2, the correction factor is 1.44.

i.e. standard uncertainty = 1.44 x estimated standard deviation

and expanded uncertainty = 2 x standard uncertainty



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