ISO BUDGETS

How to Calculate Uncertainty in Measurement Results for Calibration Certificates	by Richard Hogan, M.Eng. ISOBUDGETS LLC www.isobudgets.com October 31, 2013

Like. Connect. Follow.

About the Author

Richard Hogan is the Lead Consultant and CEO of ISO Budgets, a U.S.-based consulting company that specializes in uncertainty in measurement results and other metrology issues related to quality. Since 2002, Richard has dedicated his career to managing government and commercial laboratories in compliance with national and international standards (e.g. ANSI Z540-1, ANSI Z540.3, and ISO/IEC 17025). Richard is a U.S. Navy veteran and holds a Masters degree in Engineering Management from Old Dominion University in Norfolk, VA. He is the Virginia Section Coordinator for the National Conference for Standards Laboratories International (NCSLI), a voting member of Measurement Advisement Committee for the American Association for Laboratory Accreditation (A2LA), an active member of the American Society for Engineering Management (ASEM), and a member of the International Council on Systems Engineering (INCOSE). His research focus is in Robust Engineering, Operations Research, and Measurement Uncertainty Analysis.

Richard Hogan, M.Eng.
— President & CEO

Table of Contents

Introduction

When estimating uncertainty in measurement results related to calibration, it is important to have a process. This guide is intended to provide step-by-step instructions with brief insights and explanations to help the reader gain a greater comprehension for the task of calculating calibration uncertainty in adherence to ISO/IEC 17025:2005 and ILAC P14:01/2013.

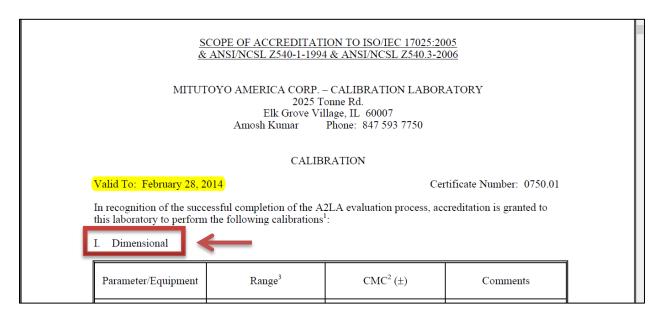The steps required to estimate and calculate uncertainty in calibration results is as follows;

01 | Identify Function/Parameter

02 | Identify Range and CMC

03 | Calculate CMC Uncertainty

04 | Quantify UUT Resolution Uncertainty

05 | Quantify UUT Repeatability Uncertainty

06 | Reduce to Standard Uncertainty

07 | Calculate Combined Uncertainty

08 | Calculate Expanded Uncertainty

09 | Report Expanded Uncertainty

01 | Identify Function/Parameter

To calculate calibration uncertainty, you will need to obtain a copy of your scope of accreditation. If your laboratory is accredited, this should not be a problem.

First, identify what function, parameter, or measurement discipline requires estimation of uncertainty.

SCOPE OF ACCREDITATION TO ISO/IEC 17025:2005
& ANSI/NCSL Z540-1-1994 & ANSI/NCSL Z540.3-2006

MITUTOYO AMERICA CORP. – CALIBRATION LABORATORY
2025 Tonne Rd.
Elk Grove Village, IL 60007
Amosh Kumar   Phone: 847 593 7750

CALIBRATION

Valid To: February 28, 2014   Certificate Number: 0750.01

In recognition of the successful completion of the A2LA evaluation process, accreditation is granted to this laboratory to perform the following calibrations[1]:

I.   Dimensional

Parameter/Equipment	Range[3]	CMC[2] ($\pm$)	Comments

Parameter/Equipment	Range[3]	CMC[2] ($\pm$)	Comments
Flatness	Up to 12 in diameter Up to 300 mm diameter	2.0 μin 0.050 μm	Comparison to master optical flat under monochromatic light source
Gage Blocks[4] –			
Length	Up to 4 in Up to 100 mm (5 to 20) in (100 to 500) mm	$(1.3 + 0.80L)$ μin $(0.033 + 0.0008L)$ μm $(1.0 + 1.0L)$ μin $(0.025 + 0.001L)$ μm	Electromechanical comparison to master gage blocks
Central Length Difference	Up to 2 in Up to 50 mm (2 to 4) in (50 to 100) mm	0.60 μin 0.015 μm 0.80 μin 0.020 μm	Electromechanical comparison between gage block pairs

02 | Identify Range and CMC

Now that the measurement discipline of interest has been selected, it is time to identify the range of that measurement function and the Calibration Measurement Capability (CMC) associated with the selected range.

Parameter/Equipment	Range[3]	CMC[2] ($\pm$)	Comments
Flatness	Up to 12 in diameter Up to 300 mm diameter	2.0 μin 0.050 μm	Comparison to master optical flat under monochromatic light source
Gage Blocks[4] – Length	**1** Up to 4 in Up to 100 mm (5 to 20) in (100 to 500) mm	**2** $(1.3 + 0.80L)$ μin $(0.033 + 0.0000L)$ μm $(1.0 + 1.0L)$ μin $(0.025 + 0.001L)$ μm	Electromechanical comparison to master gage blocks
Central Length Difference	Up to 2 in Up to 50 mm (2 to 4) in (50 to 100) mm	0.60 μin 0.015 μm 0.80 μin 0.020 μm	Electromechanical comparison between gage block pairs

03 | Calculate CMC Uncertainty

In most cases, CMC uncertainties are published or advertised as a first-order regression (i.e. linear) equation. The equation provides an output, y, quantity for a given range based on an independent variable, x, and a set of sensitivity coefficients, β_i.
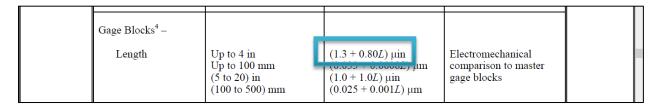
$$y = \beta_0 + \beta_1 x + \varepsilon$$

$$\int_{Min}^{Max} U_{cmc} = y = \beta_0 + \beta_1 x + \varepsilon$$

Where,
U_{cmc} = CMC Uncertainty
β_0 = y-intercept or zero offset
β_1 = slope or gain sensitivity coefficient
x = independent variable or input quantity
y = dependent variable or output quantity
ε = error

For example, if observing Mitutoyo's A2LA Scope of Accreditation (Cert# 0750.01) for 'Gage Block' calibration in the 'Up to 4 in' range, a linear equation is provided to characterize the CMC uncertainty for the given range.

	Gage Blocks[4] –				
	Length	Up to 4 in Up to 100 mm (5 to 20) in (100 to 500) mm	(1.3 + 0.80*L*) μin (0.033 + 0.0002*L*) μm (1.0 + 1.0*L*) μin (0.025 + 0.001*L*) μm	Electromechanical comparison to master gage blocks	

Using this equation, the CMC uncertainty can be quantified by the following equation;

$$U_{cmc} = y = \beta_0 + \beta_1 x + \varepsilon = 1.3 \mu in + 0.80 \mu in/in \cdot L + \varepsilon$$

Where,
U_{cmc} = CMC Uncertainty
β_0 = y-intercept or zero offset; 1.3 μin
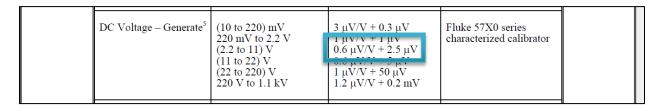β_1 = slope or gain sensitivity coefficient; 0.80 μin/in
x = independent variable or input quantity; L
y = dependent variable or output quantity; U_{cmc}
ε = error
L = Length

Similarly, the same method can be applied to Fluke Calibration's Everett Service Center A2LA Scope of Accreditation (Cert # 2166.01) to calculate CMC uncertainty for a DC Voltage signal generated in the 2.2 to 11-volt range.

	DC Voltage – Generate[5]	(10 to 220) mV 220 mV to 2.2 V (2.2 to 11) V (11 to 22) V (22 to 220) V 220 V to 1.1 kV	3 µV/V + 0.3 µV 1 µV/V + 1 µV 0.6 µV/V + 2.5 µV 0.6 µV/V + 5 µV 1 µV/V + 50 µV 1.2 µV/V + 0.2 mV	Fluke 57X0 series characterized calibrator	

$$U_{cmc} = y = \beta_0 + \beta_1 x + \varepsilon = 0.6\mu V + 2.5\mu V/V \cdot E + \varepsilon$$

Where,
U_{cmc} = CMC Uncertainty
β_0 = y-intercept or zero offset; 0.6 µV
β_1 = slope or gain sensitivity coefficient; 2.5 µV/V
x = independent variable or input quantity; E
y = dependent variable or output quantity; U_{cmc}
ε = error
E = Voltage

To calculate the CMC uncertainty using these equations, simply solve for the value of y by inputting the desired value of x into the equation.

Where L = 4 in,
$$U_{cmc} = 1.3\mu in + 0.80\mu in/in \cdot (4\ in) + \varepsilon = 4.5\mu in$$

Where E = 10 V,
$$U_{cmc} = 0.6\mu V + 2.5\mu V/V \cdot (10\ V) + \varepsilon = 25.6\mu V \approx 26\mu V$$

04 | Quantify UUT Resolution Uncertainty

Determining UUT resolution is not a difficult task. However, resolution is a factor that is often overlooked when evaluating measurement quality and reporting measurement results. Typically, it is best to report results to the value with the least number of significant digits or the worst, observable resolution.

For example, if one were to calibrate a Fluke 289 multimeter at 1 VDC using a Fluke 5720A calibrator, then the result should be reported to the value with the least number of significant digits.

> Applied by 5720A: 1.000000 V
> Measured by 289: 1.000 V
> The result: 1.000 V
> The resolution: 0.001 V
> The resolution uncertainty: 0.0005 V

Typically, for digital devices that round the least significant digit (LSD) (e.g. digital multimeter) or for analog scales (e.g. micrometer, manometer, etc.), the resolution uncertainty will be one-half of the least significant digit or scale division.

$$U_{RES} = \frac{1}{2} \cdot R = \frac{R}{2}$$

Resolution uncertainty associated with analog devices can be better than half of the LSD, but it generally requires additional apparatuses (e.g. magnifying glass) and expertise to make the determination. If one is unsure, it would best to apply the half of the LSD method.

For digital devices (e.g. stopwatches) that do not round the least significant digit, the resolution uncertainty will be the least significant digit.

$$U_{RES} = R$$

In some cases, a UUT will not have a display or scale (e.g. gauge block) which to deduce resolution. Therefore, it is common to consider the resolution of the system (e.g. CMM) which performs the measurement of the UUT. If the resolution of the system was previously included in the evaluation to quantify the CMC, then it should not be considered twice.

$$U_{RES} = R_{STD}$$

05 | Quantify UUT Repeatability Uncertainty

ILAC P14:01/2013, Section 6.4

"*Contributions to the uncertainty stated on the calibration certificate shall include relevant short-term contributions during calibration and contributions that can reasonably be attributed to the customer's device.*"

From my observations, UUT repeatability is the most commonly overlooked contributor to estimating the uncertainty in measurement results. Every measurement system is unique. This includes the new system which is created each time a new UUT is introduced.

The best method to quantify UUT repeatability is to perform a small sample (e.g. 3, 5, or 10) of repeated measurements and calculate the standard deviation of the sample set.

$$U_{RPT} = \sigma = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}}$$

06 | Reduce to Standard Uncertainty

JCGM 200:2012

"**2.30 standard measurement uncertainty**/*standard uncertainty of measurement/standard uncertainty -*
measurement uncertainty *expressed as a standard deviation*"

Before combining independent uncertainty contributors, the values must be reduced to standard
uncertainty; one-sigma (1σ) or 68.27% probability of occurrence.

To adequately reduce uncertainty contributors to standard uncertainty, the contributor must be
characterized by a probability distribution which accurately describes the behavior of the sample set.

According to the Central Limit Theorem, the CMC uncertainty will be characterized by a Gaussian
distribution and reduced from 2σ (i.e. p=95.45%) to 1σ (i.e. p=68.27%) by dividing the calculated value
by two.

$$u_{CMC} = \frac{U_{cmc}}{k} = \frac{U_{95}}{2}$$

The uncertainty associated with UUT resolution will be characterized by a Uniform (i.e. rectangular)
distribution and reduced to standard uncertainty by dividing the quantified value by the square-root of
three.

$$u_{RES} = \frac{U_{res}}{\sqrt{3}}$$

Typically, UUT repeatability will be characterized by a Gaussian distribution and reduced to standard
uncertainty by divided the quantified value by the square-root of the number of collected samples. The
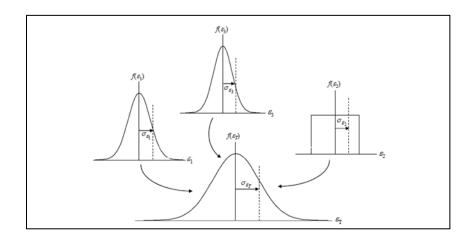result will be the calculated standard deviation of the mean.

$$u_{RPT} = \frac{\sigma}{\sqrt{n}}$$

07 | Calculate Combined Uncertainty

With all contributors to uncertainty at the same level, it is acceptable to combine the values using the root sum of squares method.

$$u_c(y) = \sqrt{u_{cmc}^2 + u_{res}^2 + u_{rpt}^2}$$

Where,
$u_c(y)$ = combined uncertainty

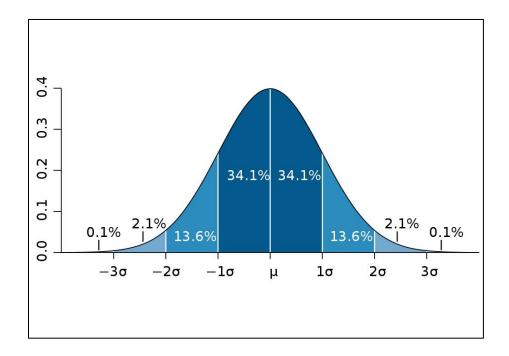## 08 | Calculate Expanded Uncertainty

Once the contributors have been combined, they will need to be expanded to a probability where k equals two (i.e. p=95.45%).

$$U = k \cdot u_c(y)$$

Where,
U = expanded uncertainty
k = expansion or coverage factor

09 | Report Expanded Uncertainty

ILAC P14:01/2013, Section 6.2

"The measurement result shall normally include the measured quantity value y and the associated expanded uncertainty U. In calibration certificates the measurement result should be reported as y ± U associated with the units of y and U."

When reporting measurement uncertainty, it is not acceptable to report equations or the CMC, identified in the laboratory's scope of accreditation, on the calibration certificate. The estimated uncertainty associated with each measurement result should be reported independently on the calibration certificate.

$$y \pm U$$

$$1.000 \, V \pm 0.00076 \, V$$

$$4.0000000 \, in \pm 0.0000046 \, in$$

FLUKE ®	Certificate Number: 673712-7845012:1374306959		Calibration Date: 20-Jul-13	
Calibration Results				
Function/Range	Nominal Value	Measured Value	Measurement Uncertainty	Manufacturer's Specifications
				Lower Limit	Upper Limit
0.000000 V	0.000000	0.0000009	5.8e-007 V	-0.0000020	0.0000020
1.000000 V	1.000000	1.0000004	3.5e-006 V	0.9999890	1.0000110
-1.000000 V	-1.000000	-1.0000009	3.5e-006 V	-1.0000110	-0.9999890
3.290000 V	3.290000	3.2900007	9.3e-006 V	3.2899684	3.2900316
-3.290000 V	-3.290000	-3.2899991	9.3e-006 V	-3.2900316	-3.2899684
33 V Range					
0.00000 V	0.00000	-0.000005	4.4e-006 V	-0.000020	0.000020
10.00000 V	10.00000	10.000006	2.7e-005 V	9.999880	10.000120
-10.00000 V	-10.00000	-10.000032	2.8e-005 V	-10.000120	-9.999880
32.90000 V	32.90000	32.900031	1.0e-004 V	32.899651	32.900349
-32.90000 V	-32.90000	-32.900053	1.0e-004 V	-32.900349	-32.899651

ILAC P14:01/2013, Section 6.3

"*The numerical value of the expanded uncertainty shall be given to, at most, two significant figures.*"

The term, significant figures, often confuses people since the term is closely associated with significant digits. The two terms are not equivalet. Significant figures are any digit of a number that is known with certainty and used to express it with a degree of accuracy, starting with the first nonzero digit.

According to Columbia University's Center for New Media Teaching and Learning (CCNMTL), there are eight rules which should be applied to the use of significant figures;

RULES FOR SIGNIFICANT FIGURES

1. **All non-zero numbers ARE significant.** The number 33.2 has THREE significant figures because all of the digits present are non-zero.

2. **Zeros between two non-zero digits ARE significant.** 2051 has FOUR significant figures. The zero is between a 2 and a 5.

3. **Leading zeros are NOT significant.** They're nothing more than "place holders." The number 0.54 has only TWO significant figures. 0.0032 also has TWO significant figures. All of the zeros are leading.

4. **Trailing zeros to the right of the decimal ARE significant.** There are FOUR significant figures in 92.00.

92.00 is different from 92: a scientist who measures 92.00 milliliters knows his value to the nearest 1/100th milliliter; meanwhile his colleague who measured 92 milliliters only knows his value to the nearest 1 milliliter. It's important to understand that "zero" does not mean "nothing." Zero denotes actual information, just like any other number. You cannot tag on zeros that aren't certain to belong there.

5. **Trailing zeros in a whole number with the decimal shown ARE significant.** Placing a decimal at the end of a number is usually not done. By convention, however, this decimal indicates a significant zero. For example, "540." indicates that the trailing zero IS significant; there are THREE significant figures in this value.

6. **Trailing zeros in a whole number with no decimal shown are NOT significant.** Writing just "540" indicates that the zero is NOT significant, and there are only TWO significant figures in this value.

7. **Exact numbers have an INFINITE number of significant figures.** This rule applies to numbers that are definitions. For example, 1 meter = 1.00 meters = 1.0000 meters = 1.000000000000000000 meters, etc.

So now back to the example posed in the Rounding Tutorial: *Round 1000.3 to four significant figures.* 1000.3 has five significant figures (the zeros are between non-zero digits 1 and 3, so by rule 2 above, they are significant.) We need to drop the final 3, and since 3 < 5, we leave the last zero alone. so 1000. is our four-significant-figure answer. (from rules 5 and 6, we see that in order for the trailing zeros to "count" as significant, they must be followed by a decimal. Writing just "1000" would give us only one significant figure.)

8. **For a number in scientific notation: N x 10^x, all digits comprising N ARE significant by the first 6 rules; "10" and "x" are NOT significant.** 5.02 x 10^4 has THREE significant figures: "5.02." "10 and "4" are not significant.

Rule 8 provides the opportunity to change the number of significant figures in a value by manipulating its form. For example, let's try writing 1100 with THREE significant figures. By rule 6, 1100 has TWO significant figures; its two trailing zeros are not significant. If we add a decimal to the end, we have 1100., with FOUR significant figures (by rule 5.) But by writing it in scientific notation: 1.10 x 10^3, we create a THREE-significant-figure value.

To simplify the application of reporting uncertainty in measurement results to two significant figures, observe the following examples;

$$346 \text{ nV} \approx 0.35 \text{ μV}$$
$$3.46 \text{ μV} \approx 3.5 \text{ μV}$$
$$34.6 \text{ μV} \approx 35 \text{ μV}$$
$$346 \text{ V} \approx 3.5 \cdot 10^2 \text{ V or } 3.5E^2 \text{ V}$$

ILAC P14:01/2013, Section 6.5

"*As the definition of CMC implies, accredited calibration laboratories shall not report a smaller uncertainty of measurement than the uncertainty of the CMC for which the laboratory is accredited.*"

Following this guide will prevent the mistake of reporting uncertainty in measurement results less than the uncertainty of the CMC published in a laboratory's scope of accreditation. With the exception of gross errors, the inclusion of the additional uncertainty contributors required by the ILAC P14:01/2013 policy should prevent noncompliance to section 6.5. Furthermore, it should be noted that a laboratory should not report estimates of uncertainty in measurement results that are less than the values published in BIPM's Key Comparison Database (KCDB).

References

Columbia Center for New Media Teaching and Learning. (n.d.). *Significant Figures*. Retrieved October 30, 2013, from Columbia University: http://ccnmtl.columbia.edu/projects/mmt/frontiers/web/chapter_5/6665.html

International Laboratory Accreditation Cooperation. (2013). *ILAC-P14:01/2013: ILAC Policy for Uncertainty in Calibration.* Silverwater: ILAC.

Joint Committee for Guides in Metrology. (2008). *JCGM 100:2008 - Evaluation of Measurement Data: Guide to the Expression of Uncertainty in Measurement.* Sèvres: BIPM.

Joint Committee for Guides in Metrology. (2012). *JCGM 200:2012 - International Vocabulary of Metrology.* Sèvres: BIPM.

Mendenhall, W. &. (2007). *Statistics for Engineering and the Sciences.* Upper Saddle River: Pearson.