Determining the Optimal Calibration Interval for Process Control Instruments

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Abstract

The paper discusses a class of stochastic models for evaluating the optimal calibration interval in measuring instruments devoted to assess the quality levels of an industrial processes. The model is based on the assumption that the calibration condition of the instrument can be traced by monitoring the drift of an observable parameter. Various stochastic drift models are introduced and compared. A preliminary validation of the model is reported, based on experimental data collected on a class of instruments.

Key words: *Stochastic process, drift models, calibration intervals, process control.*

1 Introduction

The assessment of the correct calibration conditions of measuring instruments devoted to monitoring the quality levels of an industrial process is a very crucial problem, particularly for a high technology company [1] whose quality requirements are more stringent.

The calibration of an instruments is monitored by performing periodic tests on standardized and certified specimens. Since the calibration tests require to suspend the production process, the estimation of the proper test interval is an important specification in any quality assurance programs. However, there is a surprising lack of well established and recommended methods in the international standards [2, 1]. The methods surveyed in [1] are mostly of statistical nature and can be correctly applied to large inventories of instruments, only.

The paper proposes to resort to a stochastic model that can be tailored to a single equipment. The method consists in modeling the drift of an observable parameter of the instrument, whose possible variation is bounded by a predefined tolerance level, by means of a stochastic process in time. The calibration interval is finally related to the distribution function of the first passage time of the drift process across the assigned tolerance threshold.

2 Stochastic Drift Models

The calibration condition of a given instrument can be determined by measuring the deviation of an observable parameter with respect to a preassigned value assumed as the correct one. The measured deviation undergoes a drift that can be represented by a stochastic model [3].

Let $z(t)$ be the value of the measured parameter at time t. The drift in the parameter value is supposed to be caused by a sequence of discrete random shocks whose amplitude is a stochastic variable x with known cumulative distribution function (cdf) $F_x(x)$ [4]. $z(t)$ is related to the sequence of shocks by some suitable functional. Different hypotheses on the functional dependence of the total drift on the single shock give rise to different stochastic models. Let a be a simmetric bilateral tolerance threshold on the measured parameter $z(t)$. The instruments is considered out of tolerance when the total deviation $z(t)$ exceeds the level $+\,a$ or $-\,a$.

Let T be the time at which the drift process reaches the value $\pm\,a$ for the first time.

$$T \;=\; min\,\{\, t \,:\, |z(t)| \,<\, a\,\} \tag{1}$$

T is a random variable with cdf $H_T(t) = Pr\{T \leq t\}$ and survival function $\overline{H}_T(t) = 1 - H_T(t)$.

Given a confidence level α (e.g. $\alpha = 0.95$) the calibration interval θ is defined by

$$H_T(\theta) \;=\; 1 - \alpha \tag{2}$$

We examine four cases by combining two models on the nature of the accumulation of the drift with two models on the sequence of the shocks. Specifially, the drift can be without memory (single shock) or additive, and the sequence of the successive shocks can be equispaced or random.

The drift process is without memory if the subsequent shocks are uncorrelated and do not have any cumulative effect. Hence, the total drift $z(t)$ is equal to the value of the last shock, and the instrument goes out of tolerance when a single shock exceeds the threshold $\pm\,a$. In the additive model the effect of the subsequent shocks add and the total drif $z(t)$ is equal to the sum of the shock amplitudes. The threshold is reached when the sum of the successive shock amplitudes exceeds the threshold $\pm\,a$.

The object of the subsequent analysis is to derive the survival function $\overline{H}_T(t)$ in the different cases, so that the calibration interval θ can be evaluated from (2).

2.1 Equispaced shocks

The parameter x is measured at equispaced intervals Δt.

Non-cumulative drift

The measured deviations are uncorrelated. Hence, $z(k\,\Delta t) = x_k$ and the out of tolerance condition is reached only when a single deviation x occurs with amplitude greater than $\pm a$. The probability of surviving the first k intervals is given by:

$$\overline{H}_T(k\,\Delta t) = Pr\{T > k\,\Delta t\} = Pr\{|x_k| < a,\, |x_{k-1}| < a,\, \ldots,\, |x_1| < a\} \tag{3}$$

If the deviations are independent with the same distribution, Equation (3) becomes:

$$\overline{H}_T(k\,\Delta t) = [F_x(-a, a)]^k = [F_x(a) - F_x(-a)]^k \tag{4}$$

and $\overline{H}_T(0) = 1$

Additive drift

Let us introduce the random variable s_k representing the cumulative drift up to the k-test:

$$z(k\,\Delta t) = s_k = \sum_{i=1}^{k} x_i \tag{5}$$

With the above notation:

$$\overline{H}_T(k\,\Delta t) = Pr\{|s_k| < a,\, |s_{k-1}| < a,\, \ldots,\, |s_1| < a\} \tag{6}$$

2.2 Random Shocks

The drift process is caused by shocks occurring randomly in time according to a point process $N(t)$. The amplitude of each shock is a random variable x of cfd $F_x(x)$. The process is completely specified if the probability $P_k(t)$ of having k shocks at time t is known.

If the sequence of the random shocks in time is supposed to occur according to a Poisson process of rate λ, we have:

$$P_k(t) = Pr\{N(t) = k\} = e^{-\lambda t}\frac{(\lambda t)^k}{k!} \tag{7}$$

The survival probability becomes, in this case:

$$\overline{H}_T(t) = \sum_{k=0}^{\infty} \overline{H}_T(t\,|\,k) \cdot Pr\{N(t) = k\} = \sum_{k=0}^{\infty} \overline{H}_T(t\,|\,k) \cdot e^{-\lambda t}\frac{(\lambda t)^k}{k!} \tag{8}$$

Where $\overline{H}_T(t\,|\,k)$ is the survival probability at time t conditioned on the occurrence of k shocks in $0 - t$, and is derived from Equation (3) for the non-cumulative drift and from Equation (6) for the additive drift.

An interesting result derived from [4] is that under the only hypothesis that x is positive, the survival probability $\overline{H}_T(t)$ is *Increasing Hazard Rate in Average (IHRA).*

3 Experimental Results

In order to have a preliminary validation, we have collected a sample of test data cumulated over three similar equipments whose calibration can be defined by observing the deviation of a single parameter. A test of the calibration condition is performed every work shift (three times a day). The test consists in measuring the deviation of the actual observable parameter with respect to the nominal value and reporting the measured deviation on a control chart. When the operator finds the instrument out of the calibration range, a suitable adjustment procedure is initiated.

Since the considered parameter is a deviation, the correct value should be 0, and the calibration threshold is assumed simmetric and bilateral. We have derived from the control charts of each instrument the measured deviations over a given period of time (2 months).

We have submitted the sample of the deviations to a spectral analysis . The result of this analysis is that the deviations have a white spectrum showing that there is no correlation between successive measures. Therefore, the successive measured values are not correlated and the non-cumulative drift model at equispaced time intervals seems the more appropriate.

The collected deviations have been fitted by a best-fit gaussian density (with mean value $\mu = -0.55$ and standard deviation $\sigma = 1.1$).

From the control charts we have derived the observed time between two successive out of tolerance conditions (measured in number of work shifts), and the observed empirical survival function has been fitted with the model of Equation (3) using the derived gaussian distribution as the cdf of each single shock. The agreement resulted to be satisfactorily accurate.

Finally, we have calculated the optimal calibration interval θ as the time at which an assigned percentile of the survival function of the first passage time reaches the preassigned tolerance level with three increasing values of the confidence α (e.g. $\alpha = 0.9, 0.95, 0.99$).

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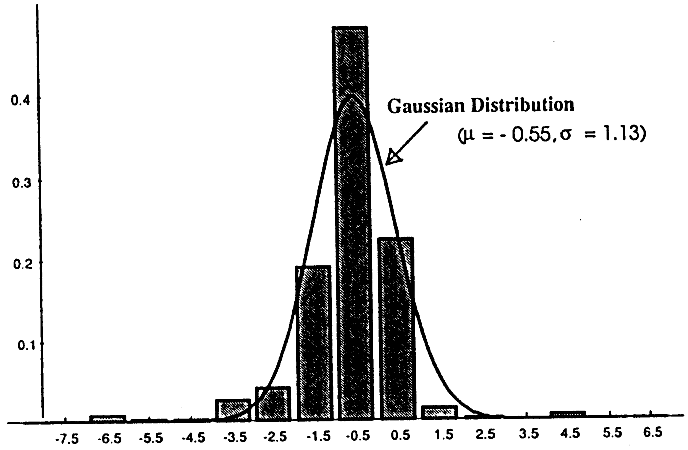

Figure 1 - Histogram of the measured deviation sample with superimposed (in solid line) the best fit gaussian density $(-0.55, 1.1)$.

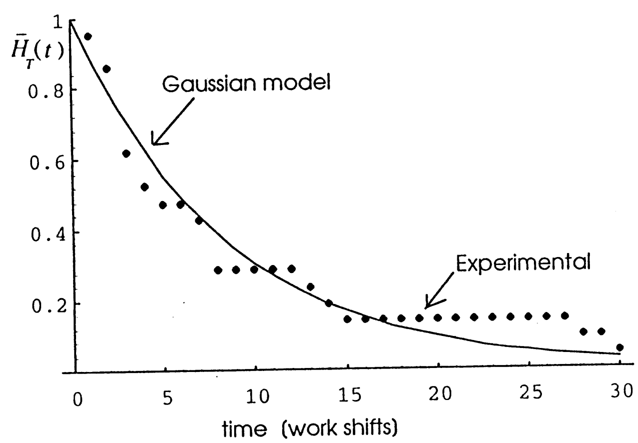

Figure 2 - The empirical cdf of the times between successive out of tolerance condition, and the sf calculated from the drift model with equispaced uncorrelated shocks.

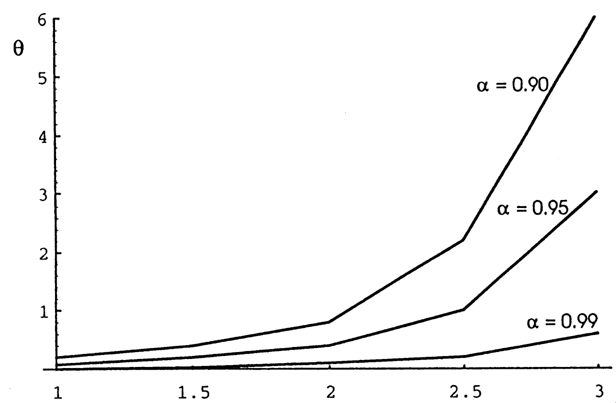

Figure 3 - The α-th percentile of the survival function versus the tolerance threshold a.